Probability Formulas and Methods

Section 14.2-14.3

Dr. John Ehrke
Department of Mathematics

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Three Probability Problems

(1) **Addition Rule Problems**

(a) *Keyword*: "or"

(b) *Formula (Case 1)*: $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ (connected sets)

(c) *Formula (Case 2)*: $P(A \cup B) = P(A) + P(B)$ (disjoint sets)

(d) *Techniques*: Venn Diagrams, Probability Tables

(2) **Complement Law Problems**

(a) *Keywords*: "none, not, at least one, at most one"

(b) *Formula*: $P(A) = 1 - P(A')$

(3) **Multiplication Rule**

(a) *Keywords*: "and, both, given that"

(b) *Formula (Case 1)*: $P(A \cap B) = P(A) \cdot P(B)$ (independent events)

(c) *Formula (Case 2)*: $P(A \cap B) = P(A) \cdot P(B|A)$ (dependent events)

(d) *Formula*: $P(B|A) = \dfrac{P(A \cap B)}{P(A)}$ (conditional probability)

(e) *Techniques*: Probability Trees

Which Formula to Use?

Example

Suppose cards are dealt from a standard deck of 52 cards. Find the probability of each of the events listed below.

(a) A single card drawn is a club or an ace.

Solution: There are 13 clubs and 4 aces in a deck. Because the problem says "or" we use the addition rule formula, and because a club can be an ace we use the connected formula. Our answer is

$$P(\text{club or ace}) = P(\text{club}) + P(\text{ace}) - P(\text{club and ace}) = \frac{13}{52} + \frac{4}{52} - \frac{1}{52} = \frac{4}{13}.$$

(b) A single card drawn is not a jack.

Solution: The use of the word "not" suggests we try using the complement law. We are asked to find $P(\text{no jack})$ but instead we will find $1 - P(\text{jack})$. Since there are four jacks in a standard deck our answer is

$$P(\text{no jack}) = 1 - P(\text{jack}) = 1 - \frac{4}{52} = \frac{48}{52}.$$

From Venn Diagrams to Probability Tables

Example

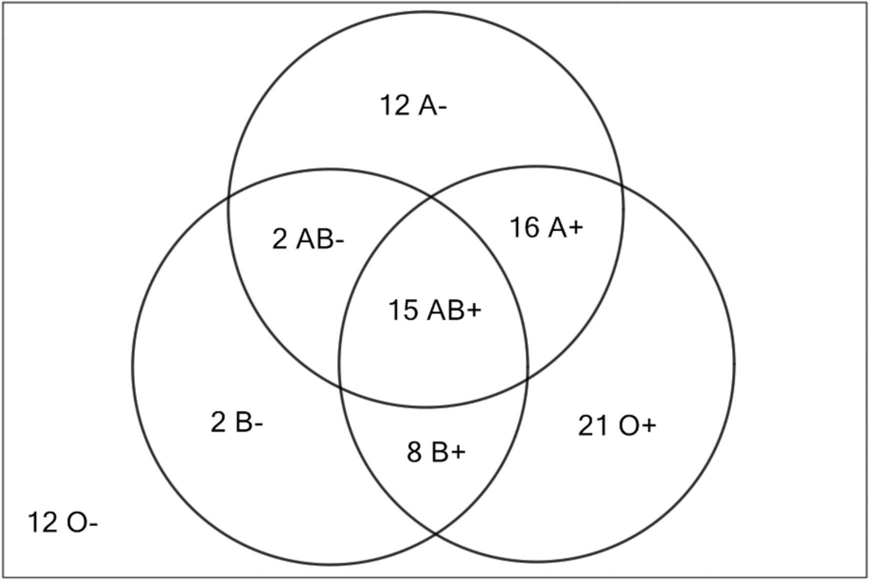
Human blood can contain either no antigens, the A antigen, the B antigen, or both the A and B antigens. A third antigen, called the Rh antigen, is significant in human reproduction, and again may or may not be present in an individual. Blood is called type A-positive if the subject has the A and Rh, but not the B antigen. Subjects having only the A and B antigens are said to have type AB-negative blood. Subjects having only the Rh antigen have type O-positive blood, etc...

Suppose we are given the following patient data on blood types at a local hospital.

Blood Antigen Data = $\{$45 A, 17 AB, 27 B, 8 B+, 60 Rh, 12 no antigens, 16 A+, 15 all antigens$\}$

Problem: Take this data and create a Venn diagram of the blood type breakdowns. Use your Venn diagram to create a table of data for each blood type.

Venn Diagram



Probability Table

Organizing the data from the Venn diagram into a table allows us to easily access the data. For example, from the Venn diagram we can tell there were exactly 88 patients tested.

	A	B	AB	O	Total
Rh +	16	8	15	21	60
Rh -	12	2	2	12	28
Total	28	10	17	33	88

Table: Blood Type Data

By dividing every term in the table by the total 88, we can turn this table into a probability table.

	A	B	AB	O	Total
Rh +	0.1818	0.0909	0.1705	0.2386	0.6818
Rh -	0.1364	0.0227	0.0227	0.1364	0.3182
Total	0.3182	0.1136	0.1932	0.375	1

Table: Blood Type Probability Data

Polling Question #10

Magnetic resonance imaging is an accepted noninvasive test to evaluate changes in the cartilage in joints. An experiment compared the results of MRI evaluations of cartilage at two tear sites in the knees of 35 patients. The total 70 examinations were classified in the table below. Actual tears were confirmed by arthroscopic examination.

	Tears	No Tears	Total
MRI positive	27	0	27
MRI negative	4	39	43
Total	31	39	70

What is the probability that a tear site selected at random has a tear or the MRI is negative?

(a) 4/70

(b) 74/70

(c) 70/70

(d) 31/70

Two Ways to the Same Answer

A group of 5 freshman, 5 sophomores, 5 juniors, and 5 seniors were selected to the student senate. A three member subcommittee will be formed to preside over the senate. If these three members are chosen at random, what is the probability that at least one is a freshman? **Find this probability in two different ways!**

Method 1: The complementary problem is the probability that no freshmen are selected. Since there are 20 students to choose for 3 positions there are $\binom{20}{3}$ total ways of selecting a three member committee. Those groups with no freshmen can be counted as $\binom{15}{3}$, so the total with at least one freshman is

$$\binom{20}{3} - \binom{15}{3} = 685.$$

Method 2: We can count each case. There is exactly one freshman, exactly two freshmen, or all three committee members are freshmen. This gives

$$\binom{5}{1}\binom{15}{2} + \binom{5}{2}\binom{15}{1} + \binom{5}{3} = 685.$$

Thinking About Conditional Probabilities

Example

A local car garage employs two mechanics, Arnie (A) and Burt (B). On a typical week Arnie does twice as many repair jobs as Burt. According to the Better Business Bureau, Arnie does a good job on his repair three out of four times, and Burt does a good job only two out of five times. If you were to randomly choose one of the two mechanics to repair your car, find the probability that a good job will be done.

Solution: The best way to handle this problem is with a probability tree (next slide). To find the probability of a good job being done we must calculate (letting G represent a good job),

$$P(A \cap G) + P(B \cap G)$$

since either Burt (B) or Arnie (A) can do a good job. In each case we have

$$P(A \cap G) = P(A) \cdot P(G|A) = \frac{2}{3} \cdot \frac{3}{4} = \frac{1}{2}, \qquad P(B \cap G) = P(B) \cdot P(G|B) = \frac{1}{3} \cdot \frac{2}{5} = \frac{2}{15}.$$

So our answer is

$$P(A \cap G) + P(B \cap G) = \frac{1}{2} + \frac{2}{15} = \frac{19}{30}.$$

Probability Tree

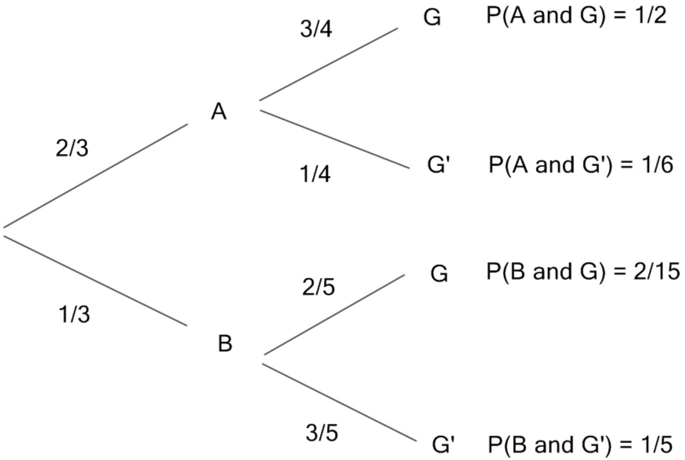


Figure: Probability Tree For Conditional Probabilities

Polling Question #11

Suppose two cards are dealt in succession (without replacement) from a standard deck of 52 cards. Find the probability that the first card is a jack and the second is a face card. The face cards are jack, queen and king.

(a) $\dfrac{11}{676}$

(b) $\dfrac{4}{221}$

(c) $\dfrac{3}{169}$

(d) $\dfrac{11}{663}$

Polling Question #12

If there are 8 red disks numbered 1 through 8, and 2 yellow disks numbered 9 through 10, find the probability of selecting a red disk, given that an odd-numbered disk is selected.

(a) $\frac{4}{5}$

(b) $\frac{1}{10}$

(c) $\frac{2}{5}$

(d) $\frac{1}{5}$

Polling Question #13

The table below organizes the number of minority officers in the US military in the year 2000. Use this table to find the probability of selecting an officer who is in the Navy, given that the officer is African American.

	Army	Navy	Marines	Air Force
African Americans	9162	3524	1341	4282
Hispanic Americans	2105	2732	914	1518
Other Minorities	4075	2653	599	3823

(a) $\dfrac{3524}{18,309}$

(b) $\dfrac{8909}{18,309}$

(c) $\dfrac{3524}{8909}$

(d) $\dfrac{3524}{14,785}$

Class Exercise #1

The following problem is referred to as the birthday problem, but we will break it up into pieces. Our goal is to determine the probability that at least 2 people in a group of n people ($n \leq 365$) share a birthday. By birthday we mean month and day, not the year. We will disregard the possibility of a leap year in our calculations.

1. Step One: Find the probability that no two people have the same birthday in a group of size $n = 3$.

2. Step Two: Find the probability that no two people have the same birthday in a group of size $n = 5$. Do you see a pattern in the calculation?

3. Step Three: Use your observations from step one and step two to determine a general formula for a group of size n.

4. Step Four: When is there a 50-50 chance, i.e. the odds are 1:1, for two people sharing the same birthday?