# PHYS 1402: College Physics II

## Laboratory Manual

College of Arts and Sciences
LAMAR UNIVERSITY
Department of Physics

Compiled and edited by Dr. James Drachenberg

# Acknowledgments

This laboratory manual was compiled and edited by Dr. James Drachenberg drawing liberally from exercises developed over many years by the Lamar University Physics Department and the Department of Physics and Astronomy of Valparaiso University.

# Lab 1: Harmonic Motion and Data Analysis

## Overview

Oscillatory motion occurs in many physical systems. All swinging or vibrating objects in everyday life are examples. One particularly important type of oscillatory motion is ***simple harmonic motion***. A body moves under simple harmonic motion when the net force acting on the body is always ***directly proportional*** to the displacement of the body from its equilibrium position. Furthermore, this force is always oriented to return the body to its equilibrium position. A mass oscillating on the end of a spring is an example of simple harmonic motion. The period $T$ of a mass on a spring under simple harmonic motion is related to the size of the mass and to the spring constant $k$ by:

$$T = 2\pi\sqrt{\frac{m}{k}} \tag{1}$$

More complex oscillatory motion, e.g. a vibrating guitar string, can often be represented as the combination of several simple harmonic motions occurring simultaneously.

## Learning Objectives

- Determine how the period of the simple harmonic motion depends on the size of the suspended mass and use this to obtain an experimental value for the spring constant.
- Gain an introduction to the analysis of experimental data and techniques we will use throughout the course of the semester.

## Procedure:

### Part A – Data Collection:

In this lab, we will collect a common set of data that each lab group will analyze. Every student will take a measurement and record it in a spreadsheet to be e-mailed to the instructor. The instructor will compile all measurements from every lab group and post the full dataset on Blackboard.

Each student will take a measurement of the periods for eight different masses on the end of a spring. To obtain the highest quality period measurements, set the mass-spring system oscillating and then **wait a few cycles before beginning the data collection**. This will allow the system to reach a consistent, steady-state oscillation. Use the same number of cycles each time for your data collection. We recommend 10 cycles to get a robust average. Be sure that your counting of the first oscillation includes a full cycle through the motion. Record in a spreadsheet the average periods measured by each student for each mass. E-mail the spreadsheet to your instructor, who will compile the full set from the data from each group and post the spreadsheet to Blackboard.

### Part B – Data Analysis:

We want to extract from the full set of period data (posted on Blackboard) the best possible estimate of the spring constant, $k$. One of the most powerful tools for such a task is the ***linear regression***. The linear regression can be easily obtained utilizing the Data Analysis package in Excel. We will do this many times throughout the course of the semester.

We need to do a bit of thinking for our analysis. A few questions:

- Is Equation (1) valid for our spring-mass system?
- Can we express it in such a way as to test its validity through a linear regression?

In your group, discuss what specific quantities you can plot on the horizontal and vertical axes to test the validity of Equation (1) through a linear regression. If you do not immediately have these values, think about how you can manipulate the data to obtain them.

Manipulate, plot, and analyze the period data on your spreadsheet to determine whether the equation is obeyed. Remember that a linear plot has two parameters, e.g. a slope and a $y$-intercept. ***Pay particular attention to the intercept of the graph***.

### Part C – Interpretation of the Data:

From the results of your linear regression, determine the spring constant $k$ and the associated statistical uncertainty in your measurement. What conclusions can you draw from your results? For example, discuss the validity (or lack thereof) of Equation (1) for our spring-mass system. Are your results consistent with expectation? Are there any ***systematic errors***?

## Turn It In

1. E-mail to your instructor and your lab partners a copy of your spreadsheet with the results of this laboratory. Don't forget to include the error analyses, regression output, and associated graphs needed to obtain your results.
2. Each student should hand in an independent Lab Report page, including a discussion of the results. Don't forget to sign your name at the top of the page!

# Lab 2: Standing Waves

## Overview

In this investigation, you will examine the behavior of standing waves. Standing waves are formed when a wave traveling along one direction is reflected from an interface; the original wave and the reflected waves interfere with each other creating a new net wave pattern. If the conditions are correct, a wave that seems to be stationary is created. These "standing waves" have large amplitudes and are well-defined. They are responsible for the discrete notes played on most types of instruments.

There are two components to this lab: measuring standing waves on a taut string and measuring the sound waves (pressure waves) in air in a pipe.

## Objectives

- Observe and measure standing wave patterns on a taut string and in a pipe.
- Determine the speed of sound in air.

## Setup & Equipment

**Waves on a String:** A wave generator is plugged into the wall socket, which provides a frequency of the vibration equal to 120 Hz. A string is attached to the wave generator. The string is held under tension by attaching a weight to the other end (which is passed over a pulley). This weight can be changed, allowing the tension in the string to be changed.

**Sound Waves in a Pipe:** The apparatus includes a long PVC pipe, a tuning fork, a block (surface to strike the tuning fork), and some rubber bands. The length of the pipe is adjustable.

## Theory

As stated above, standing waves form when a wave reflects off an interface; the incoming wave interferes with the reflected wave, setting up a pattern. This reinforces certain wavelengths and is the principle behind sound generation by instruments—sounds with incorrect wavelengths are suppressed, while sounds with the correct wavelengths are amplified.

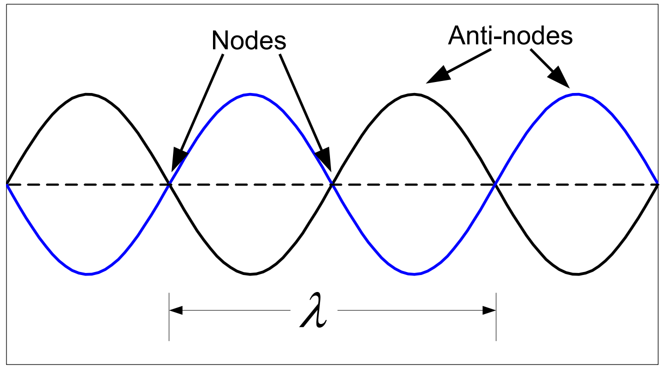



**Figure 1.** A standing wave on a string that is clamped a both ends, showing the wavelength and the definition of nodes and anti-nodes. The string vibrates betweeı the two extremes shown

For stringed instruments, the ends of the strings are attached to rigid supports, such that the string ends are relatively unable to move. Places where the string is not moving are called "nodes," while places where the string has the largest movement are called "anti-nodes" (see Fig. 1). Besides the stationary points at the ends of the strings, the wave may have zero, one, two, etc., nodes along its length. Figure 1 shows a standing wave pattern where there are 3 nodes. Notice that for this specific pattern, the wavelength of the wave is $\lambda = L/2$ where $L$ is the length of the string. Other patterns will have a different relationship between the wavelength and the length of the string.

One of the basic equations that governs waves is the relationship between speed, frequency, and wavelength:

$$v = f\lambda. \tag{1}$$

Knowing the frequency and the wavelength, you can find the speed of travelling waves using Equation (1). Recall that even for "standing waves", there are in fact two *travelling* waves moving in opposite directions.

What determines the speed at which waves travel? For a wave on a string, it turns out to be related to force applied to the string (the tension) and the density of the string. From experience, you may have some sense that waves travel faster along a string that is stretched tighter. The relationship between wave speed $v$ and tension $T_s$ is given by

$$v = \sqrt{\frac{T_S}{\mu}}. \tag{2}$$

That is, the velocity is proportional to the square-root of the tension.

## Procedures & Analyses

### A. Standing Waves on a String

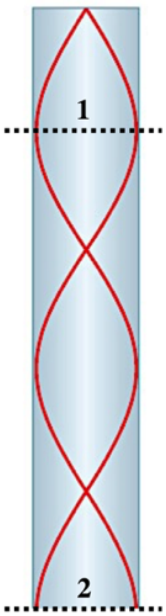
There are several hallmarks of standing waves:

1) A static pattern (there should be no movement of the nodes along the string).
2) The nodes should have essentially zero amplitude (they should not be blurry).
2) Large amplitude of the anti-nodes (the waves reinforce each other strongly).
3) The ends of the string should be good nodes (you should not see much amplitude of the string at its ends).

*Play:* Play around with the setup for a few minutes. Turn on the wave generator and apply some tension to the string by manually pulling on the weight hanger. Notice that usually no standing wave is produced, but for certain amounts of tension different patterns will show up. Similarly, you can put some tension in the string by hanging some weight. With the wave generator turned off, try plucking the string near its end. Can you tell if the speed of the wave is going to be small or large?

*Tension Dependence (constant length):* Change the tension in the string by changing the hanging weight. Find 5 different tensions that give standing waves. Be as careful as possible about when the optimum standing waves are formed, adjusting the hanging weights by as little as reasonably possible. In Excel, for each data point, record the number of nodes, $n$, and the amount of mass on the hanger. Add another column for the tension (calculated from the amount of mass). Calculate the wavelength and the speed of the wave for each data point (using Equation (1)). You should see that as the tension increases the speed increases (and so does the wavelength, since the frequency is held constant), but you should see that it is not a linear relationship (for instance, if the tension is doubled, the velocity does not double). Using Excel, make a plot of the speed versus the tension. For this portion of the lab, you do not need to perform a linear regression or add a trend line.

## B. Sound Waves in a Pipe




*Experiment:* Here, we want to parlay our intuition about waves (e.g. those on a string) into quantitative knowledge of one of the most familiar waves we encounter daily—sound. In the figure on the left, you see a depiction of a standing sound wave contained exactly within a pipe. The two horizontal lines bound a distance of precisely one wavelength. Notice in the wave depicted, there are two nodes within the pipe and one at the closed end. As a lab group, play around with the pipe/tuning fork setup for a few minutes and discuss how you might use these mundane items to measure the speed of sound. Some relevant questions you may wish to discuss are as follows: We know how to recognize a standing wave on a string, but ***how can we recognize a standing sound wave?*** While we know some physical quantities that we can use to calculate the speed of a wave, ***how can we apply our tools to get these quantities?*** Notice that you have tuning forks of varying frequency and a pipe of variable length. Don't be afraid to try crazy ideas (do take care only to strike the tuning forks on the blocks provided; and don't touch a struck tuning fork to the pipe, lest it break)!

Once in-hand, analyze your data in Excel to extract a measurement of the speed of sound. ***You will find a linearized version of Equation (1) to be quite useful!*** Perform a linear regression to get the slope, intercept, and associated uncertainties. Using this information, ***quantitatively*** compare your extracted value to the expected value, i.e. the known speed of sound. Do the same for your intercept.

## Turn In

1. E-mail to your instructor and your lab partners a copy of your "data" sheets with the results of both parts of this laboratory. Don't forget to include the error analyses, regression output, and associated graphs needed to obtain your results.
2. Each student should hand in an independent Lab Report page, including a discussion of the results. Don't forget to sign your name at the top of the page!

# Lab 3: Thin Lenses

## Overview

In this laboratory, you will have hands-on experience in measuring properties of lenses and the images they form.

## Learning Objectives

- Distinguish converging lenses from diverging lenses
- Understand image formation using lenses
- Understand the distinction between real and virtual objects and images
- To combine lenses: examining their net effect on image formation

## Introduction

A simple thin lens is usually a single circular piece of glass with one or both surfaces ground to a spherical shape. The **optical axis** of a lens is the straight line through the center of the lens, perpendicular to both faces of the lens, and through the center of curvature of the spherical surfaces. For the thin lens theory of our exercises, rays of light approach and leave the lens making small angles with the optical axis. These are called paraxial rays.

A thin lens has two **focal points**, both on the optical axis and equidistant from the center of the lens. One focal point is in front of the lens a distance equal to the focal length from the lens. The other focal point is behind the lens a distance equal to the focal length from the lens. A thin lens is thin enough to be treated as if it has zero thickness.

A **converging lens** bends rays of light toward the optical axis. If a converging lens intercepts rays of light diverging from a point source of light, the intercepted rays are bent so that they converge to a point, called the image point, or at least, are bent so that they seem to come from a point farther from the lens than the point source of light.

A **diverging lens** bends rays of light away from the optical axis. If a diverging lens intercepts rays of light diverging from a point source of light, the intercepted rays are bent so that they diverge even more and seem to come from a point between the lens and the point source of light.

**Thin Lens Equation**

The distance, measured parallel to the optical axis, from the object to the lens is called the **object distance** ($s$). The distance, measured parallel to the optical axis, from the lens to the image is called the **image distance** ($s'$). The focal length, $f$, is related to $s$ and $s'$ by the thin lens equation

$$\frac{1}{f} = \frac{1}{s} + \frac{1}{s'}. \tag{1}$$

**Magnification**

The magnification of an individual lens is determined by the ratio of the image size $h'$ to the object size $h$. Using some basic trigonometry with the thin lens equation and ray tracing, it can also be shown that the magnification is equal to the ratio of the image distance $s'$ to the object distance $s$. So, we have

$$M = \frac{h'}{h} = -\frac{s'}{s}. \tag{2}$$

**The Sign Convention**

1. The object distance ($s$) is **positive** when the light travels from the object to the lens. In this case, the object is called a **real object**. If you put your eye at the location of a real object, you would see the object. The object distance ($s$) is **negative** when the light appears to travel from the lens toward the object, now called a **virtual object**. If you put your eye at the location of a virtual object, you would ***NOT*** see the object.

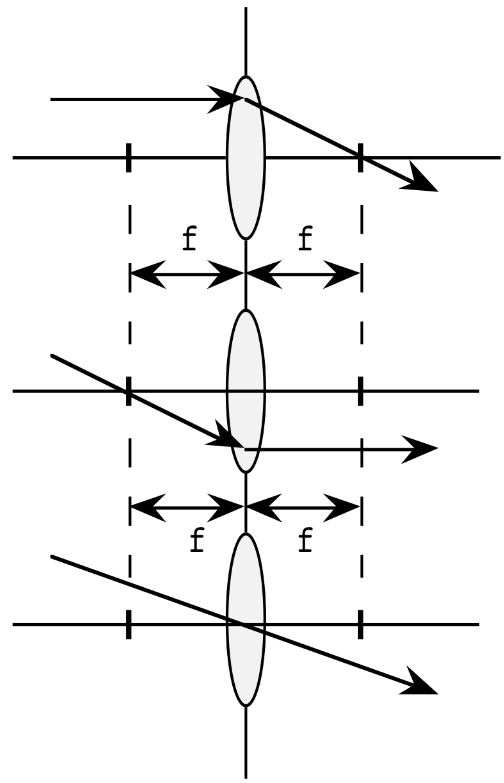
2. The image distance ($s'$) is **positive** when the light travels from the lens to the image. When the rays really come to focus at the image location, we call the image a **real image**. Placing a detecting device (such as a screen or the retina of your eye) at the location of a real image will allow the image to be seen.

3. The image distance ($s'$) is **negative** when the light seems to travel from the image, called a **virtual image,** to the lens. Placing a screen at the location of a virtual image will not result in a detectable image. A virtual image exists at the location from which light rays appear to come. In reality, the light rays do not pass through the location of a virtual image.

4. The focal length ($f$) of a converging lens is **positive** while the focal length of a diverging lens is **negative**.

**Ray Diagrams**

The rough qualities of an image can be determined using a simple set of rules about how light rays get deviated by a lens. Out of the countless numbers of rays that emanate from a point on an object (e.g. it's tip), we can easily trace 2 or 3 of them through the lens to see where they meet up, or appear to have met up in the case of a diverging lens. In practice, it's easiest to use two of the following three rays, known as principle rays:




The **parallel ray** approaches a lens parallel to the optical axis, but not on the axis. This ray leaves a converging lens passing through the second, or back, focal point.

The **focal ray** passes through the first, or front, focal point and emerges from a lens parallel to the optical axis, but not on the axis.

The **central ray** approaches the center of a thin lens, making a small angle with the optical axis, and travels through the lens with no change in direction.

**Notice:** We have three distinct points—the two focal points and the center of the lens. We also have three principal rays. One and only one principal ray passes through each of these three points.

To locate the complete image of an object we need to locate the image points of at least two points on the object, usually the top and bottom points of the object. When the object is located with one of these extremes on the principle axis of the lens (as it often is in PHYS 1402 problems), the second point is chosen to be the point of the object that is on the optical axis. The image of this point on the optical axis will also be on the optical axis. And if the object is perpendicular to the optical axis, as it is usually placed, the image will also be perpendicular to the optical axis. Therefore, in practice, the image of only one off-axis object point is often needed to locate the image for introductory physics problems.

# Lab 4: Interference and Diffraction

## Overview

The wave-nature of light becomes apparent when passing it through narrow slits, or shining it around small objects. The light will diffract (bend) and multiple sources of light will interfere with each other, creating interference patterns. These patterns can be analyzed to determine the sizes of the sources and their arrangement, or the wavelength of the light.

## Learning Objectives

- You will learn how to analyze interference patterns to determine the geometry of the source(s) of light.
- You will use diffraction and interference to study the sizes of common (but small) objects.

## Equipment:

A CD, a DVD, a laser, a screen, and some rulers.

## Procedure and Analysis:

### A. Single "Slit" Diffraction

Light shining on a narrow object will diffract around it in the same way that light diffracts when going through a rectangular slit. Obtain a hair from your head and use tape with a suitable apparatus to orient it vertically in front of the laser. You should see a pattern of high intensity and low intensity—bright and dark fringes. To get the most data possible, mark the dark fringe positions on both sides of the central maximum. The dark fringes are expected to be located at angular positions given by

$$\sin\theta_p = \frac{p\lambda}{a} \qquad p = 1, 2, 3, \dots \tag{1}$$

where $a$ is the width of the slit, $p$ is the order number (which dark fringe is it, counting out from the center), and $\lambda$ is the wavelength of the light (given on the apparatus). When projected onto a screen a distance $L$ away, dark "fringes" are seen a distance $S$ to the right (or left) of the central spot

$$S = L\tan\theta\,. \tag{2}$$

If $L \gg S,$ then we can use the small angle approximation, $\tan\theta \approx \sin\theta$, to get

$$S = L\sin\theta_p = L\frac{p\lambda}{a} \rightarrow a = p\frac{L\lambda}{S} \tag{3}$$

Determine the positions and uncertainties of a few of the dark fringes. You also may need the position of the central maximum. Record these values in Excel. Determine all the various values of $S$ you can. In Excel calculate $a$ for each value of $S$ and $p$ and therefore generate a list of values of $a$. Find the average and the standard error.

### B. Using Diffraction to Determine the Track Pitch of CDs and DVDs

When laser light passes through a diffraction grating, you see a bright central spot and a few bright spots on either side. In contrast to that for a single or double slit, a set of many slits concentrates the diffracted light into a few very bright spots rather than many dimmer fringes. The equation for the location of the bright spots is

$$d \sin\theta_m = m\lambda \qquad m = 0, 1, 2, \ldots \tag{4}$$

where $d$ is the distance between the slits and $\theta$ is the angle at which the bright spot occurs. A commonly used variable to describe diffraction gratings is the number of lines per mm, $N$,

$$N = \frac{1}{d} = \frac{\sin\theta_m}{m\lambda} \rightarrow d = \frac{m\lambda}{\sin\theta_m}. \tag{5}$$

CDs and DVDs have tracks that are closely spaced and therefore act like diffraction gratings. The spacing between the tracks is called the "track pitch." DVDs hold about 6 times more data than CDs in part because the tracks are more closely spaced. If you shine a laser onto a CD or DVD, the light will be reflected and diffracted, forming well-defined diffraction spots. Assuming the light is incident normally on the CD or DVD, by measuring the angle and knowing the wavelength, you can determine the track pitch ($d$ value) using equation (5).

Position the CD and shine the laser on the CD at normal incidence. **NOTE:** Part of the laser light reflects rather than diffracts. Adjust the tilt of the CD and the laser such that the reflected laser light bounces directly back to the laser. Use a ruler or other equipment to determine the positions of any diffraction spots you find, taking as much care as you can to get good measurements. Record your values in Excel. In your lab report, be sure to describe your measurement technique as you have a wide range of choices to make in how to perform these measurements and to perform them well.

Look at both the CD and the DVD and determine their track pitches. The DVD's track pitch should be very close to 0.74 µm, while the CD's may range from 1.5 to 1.7 µm. Perform your calculations in Excel and report your final values and uncertainties on the lab report.

## Turn It In

1. Each student should turn in a completed lab report form, including discussion of the data.
2. E-mail a copy of your data spreadsheet with the results of both parts of this laboratory. Don't forget to include anything needed for obtaining results: error analyses, regression output, associated graphs, etc.

# Lab 5: Electric Potential and Electric Field Lines

## Overview

In this laboratory, we will use an apparatus to simulate various electrostatic conductor arrangements. Equipotential lines will be mapped out, and electric field lines will then be drawn perpendicular to those equipotential lines.

## Equipment:

A voltmeter, electrodes, paper towel, Plexiglas surface, spray bottle, and grid paper.

## Procedure and Analysis:

We will use a voltmeter to determine points of constant potential (voltage) in a wet paper towel on a Plexiglas surface for some electrode configurations. Electrodes placed at specified potentials simulate charged conductors. Although an A/C (alternating current) power supply is used, the RMS (root mean square) voltage is the same as in the electrostatic case.

Take your Plexiglas sheet and paper towel to the sink and spray down the paper towel until soaking wet. Allow excess water to drain. Force out all air bubbles from under the towel as best you can. The black grid should be visible under the wet paper towel.

The instructor's table has a 10 V A/C power supply which is sent to each of the lab tables. The black lead of the voltmeter is connected to the negative electrode. *This is now the "zero" potential.* The red lead can be used to measure the potential at selected points.

Map out points for equipotential lines at 2V, 3V, 4V, 5V, 6V, 7V, and 8 V (1V and 9 V may be too close to the electrodes to measure). Plot points on the sheets of graph paper provided (identical to the plastic). Make sure for each voltage sufficient points are determined so a smooth curve may later be drawn between the points, indicating the equipotential lines.

After the equipotential lines are determined, use a different colored pencil to draw in electric field lines, being careful to ensure the field lines always cross the equipotential lines *perpendicularly.* Also, make sure the field lines are perpendicular to the electrodes (which are, of course, equipotential surfaces). It is recommended that you first SKETCH the field lines lightly in erasable pencil, and retrace in colored pencil when you get them right.

Complete the process for two configurations of electrodes, as described by your instructor. Each student should have two graph sheets showing the equipotential and field lines for the two electrode configurations. ***Turn in your graph sheets before you leave the lab.***

# Lab 6: Capacitors

## Overview

In this laboratory exercise, you will explore the behavior of capacitors and combinations of capacitors, using multimeters and a ballistic galvanometer.

## Learning Objectives

The main goal will be to determine the capacitance of capacitors and capacitor combinations by making voltage measurements and determining the charge storage with a ballistic galvanometer.

## Introduction

The capacitance of a capacitor is defined as the ratio of the charge, $Q$, stored on one plate to the potential difference, $V$, across the plates. We can, thus, express $Q$ as a function of $V$

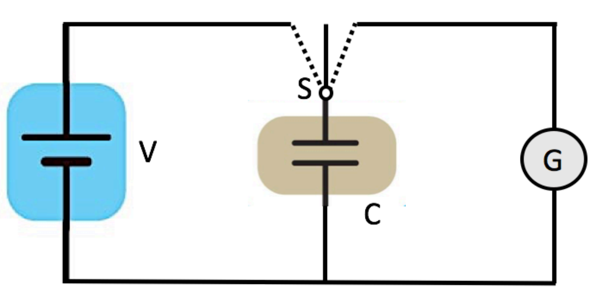
$$Q = C \times V. \tag{1}$$

You will use the following method to measure the capacitance of a capacitor: apply a known potential difference, $V$, to charge the capacitor; and, then, discharge the capacitor through a charge-meter (a ballistic galvanometer--"$G$" in the diagram, below) to measure the charge, $Q$. The capacitance will be the "rate" at which charge depends on voltage, as in equation (1).

The ballistic galvanometer on your table has already been calibrated. The constant, $k$, on the label indicates the number of micro-Coulombs needed to deflect the meter with one division. Therefore, if the meter is deflected $d$ divisions by a discharging capacitor, the charge coming off the capacitor is

$$Q = kd \tag{2}$$

Record the value of $k$ for your meter on your lab report and use it throughout the experiment.




The circuit we want to use in this experiment is shown in the schematic at left. When the switch, $S$, is thrown toward the output, a potential difference, $V$, is impressed across the capacitor, $C$. When the switch is thrown all the way to the right, the capacitor discharges through the charge meter (ballistic galvanometer, $G$).

## Part I: Individual Capacitors

Two individual capacitors will be used in this part of the experiment. The large capacitor is $C_1$, and the small capacitor is $C_2$.

- Adjust the ballistic galvanometer to zero on the left side of the scale.
- Refer to the circuit diagram. Set the switch, $S$, in the neutral (open) position.
- Connect capacitor $C_1$ at the location marked $C$ in the circuit diagram.
- Set and record the potential difference, $V$, which will be applied to the capacitor.
- Throw the switch to the contacts with the output (the charging position, the capacitor will be fully charged to the potential $V$ in less than one second).
- Discharge the capacitor by flipping the switch all the way to the other side (do ***NOT*** hesitate in the open position).
- The charge that was on the capacitor will flow through the ballistic galvanometer. Note and record the maximum deflection (in divisions, $d$) of the galvanometer.
- Repeat this process three times, using different values of $V$. Try potentials that will give a large range of deflections for the four trials, e.g. 15-25 divisions, 30-40 divisions, 45-65 divisions, and 70-80 divisions. You should wind up with four total trials for $C_1$.
- For each trial, calculate and record the charge stored on the capacitor, $Q = kd$.
- Make a plot of $Q$ vs. $V$ (including a trendline) from your data for $C_1$. Perform a linear regression to get the slope, intercept, and uncertainties. The slope should be your measurement of $C_1$, as shown in Eq. 1. Quantitatively evaluate your intercept result.
- **Repeat the above procedure for $C_2$.**

## Part II: Parallel Capacitor Combinations

Two capacitors will be connected in parallel in this part of the experiment. Again, the large capacitor is $C_1$, and the small capacitor is $C_2$.

- Utilize your extracted values for $C_1$ and $C_2$ to calculate a "theoretical" value for the parallel combination, "$C_P$," of $C_1$ and $C_2$.
- Connect $C_1$ and $C_2$ in parallel and place the combination at location $C$ in the diagram.
- Using the procedure from Part I, find the "experimental" capacitance of $C_P$.
- Quantitatively compare your two results.

## Part III: Series Capacitor Combinations

Two capacitors will be connected in series in this part of the experiment. Again, the large capacitor is $C_1$, and the small capacitor is $C_2$.

- Utilize your extracted values for $C_1$ and $C_2$ to calculate a "theoretical" value for the series combination, "$C_S$," of $C_1$ and $C_2$.
- Connect $C_1$ and $C_2$ in series and place the combination at location $C$ in the diagram.
- Using the procedure from Part I, find the "experimental" capacitance of $C_S$.
- Quantitatively compare your two results.

## Turn In

3. E-mail to your instructor and your lab partners a copy of your "data" sheets with the results of all parts of this laboratory. Don't forget to include the error analyses, regression output, and associated graphs needed to obtain your results.
4. Each student should hand in an independent Lab Report page, including a discussion of the results. Don't forget to sign your name at the top of the page!

# Lab 7: Ohm's Law and Resistance

## Overview

In this laboratory exercise, you will explore both qualitatively and quantitatively the behavior of simple, linear circuits consisting of a power supply and a set of resistors wired in both parallel and series combinations.

## Learning Objectives

The main goal will be to determine the resistance of resistors and observe Ohm's Law by making voltage and current measurements.

## Introduction

The flow of electricity best follows paths provided by materials that have a low electrical resistance. Such materials are commonly metals, and one of the best electrical conductors is copper wire.

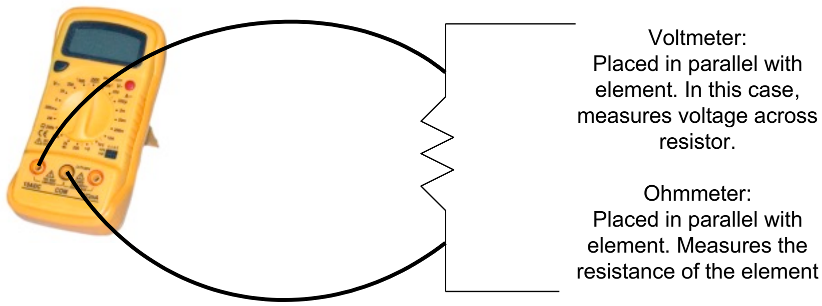
Electric current does not flow spontaneously but requires some expenditure of energy. Numerous sources can be employed to generate electrical currents: an electric cell, a battery of cells, a generator, a solar cell, etc. In some cases, this electric energy can be stored (as in a cell or battery of cells) and in other cases, the energy is supplied on demand (e.g., electric energy provided by a generator).

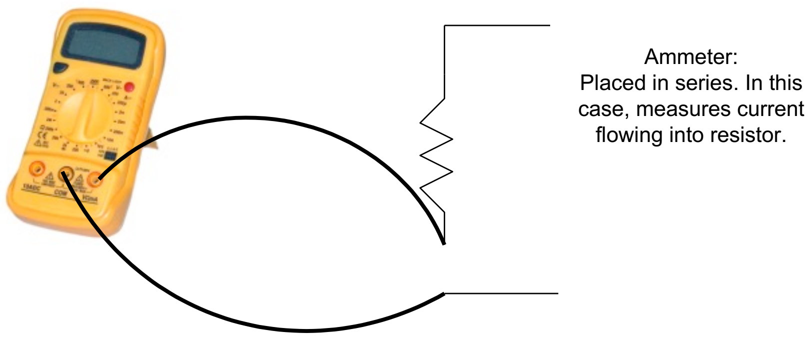
### Measuring Device

The measuring device you have is called a "multimeter." It is used to measure voltages across circuit elements, the amount of current flowing in a part of the circuit, and the resistance of a circuit element. The part that measures the voltages is called a "voltmeter," the part that measures the current is called an "ammeter," and the part that measures the resistances is called an "ohmmeter."

When you measure voltages or resistances, you will attach the multimeter in parallel to the element you're measuring. That is, one of the leads will be located on one side of the element and the other lead will be on the other side of the element. ***Note that you cannot measure the resistance if the circuit is energized***—disconnect the batteries before measuring the resistance.

When you measure current in a wire, ***you will need to place the ammeter in series with the wire***. You'll also have to change which ports on the multimeter are used.



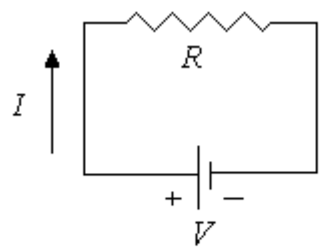




Be careful with the multimeters. If you have it wired for measuring currents, set it to ammeter, and make sure that it is not hooked up in parallel with anything in the circuit—it must be in series with the circuit element you want to measure the current through.

## Ohm's Law

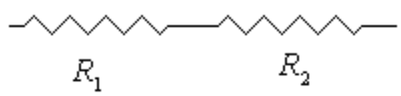
Ohm's Law relates the electrical resistance with the voltage and current:

$$\boxed{V = IR} \qquad \text{Ohm's Law}$$



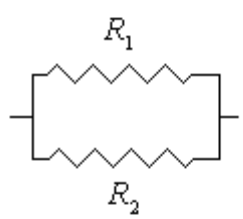

For a series arrangement of resistors, the equivalent resistance may be written

$$\boxed{R = R_1 + R_2 + R_3 + \cdots} \qquad \text{Series}$$




Similarly, for a parallel arrangement of resistors, the equivalent resistance is given by

$$\boxed{\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots} \qquad \text{Parallel}$$



# Lab 8: The Field of a Magnetic Dipole

## Introduction

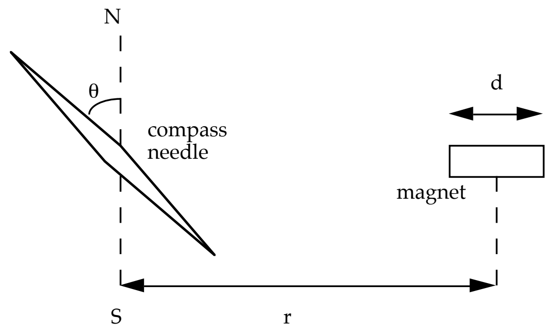
In physics, we are often interested in the variation of forces and fields as a function of distance. This tells us something of their fundamental character. One very important result in experimental physics was Coulomb's discovery in 1785 of the $1/r^2$ dependence of the electric force between two charges. In this experiment, you will use very simple apparatuses—a meter stick, a magnetic compass, and a small set of magnets—to investigate the nature of the distance-dependence of the magnetic field $B_m$ from a dipole magnet. Your intuition probably tells you that the field strength should decrease as the distance $r$ from the magnet increases. Finding a mathematical expression that describes this quantitatively is the goal of the experiment.

## Procedure

You will measure the deflection of a compass needle caused by a small dipole magnet a certain distance away. To ensure an accurate measurement of the distance, you will need to set the bar magnet to the same height above the lab bench as the compass needle (e.g. use sheets of paper as a base upon which the magnet may rest). Since the direction of the field $\vec{B}_m$ due to a dipole magnet is constant along the axis of the magnet, it is also important for the magnet axis to be exactly perpendicular to the N-S direction of the earth's magnetic field (the direction the compass needle points if it experiences no other magnetic fields). In this configuration, the deflection of the compass needle $\theta$ can be measured as the magnet is moved to different distances $r$ from the compass needle. These data, in the form of $\theta(r)$, can be used to find the dependence of $B_m$ on the distance $r$, which we will assume has the mathematical form

$$B_m = \frac{C}{r^n}, \qquad (1)$$

where $n$ and $C$ are constants. Your specific assignment is to find the value of the exponent $n$.




1. Use a convenient method to raise the bar magnet the same distance above the lab bench as the compass needle. Ensure that your method will allow moving the magnet while keeping the compass fixed in place.

2. Remove the magnet to a large distance from the compass and allow the needle to swing freely to the N-S direction of the horizontal component of the earth's magnetic field $\vec{B}_{e,h}$. Level the compass carefully, and rotate the scale so that the pointer is at 0°. Devise a way to indicate the direction exactly perpendicular to N-S, e.g. use string, tape, a meter stick, etc. Once in place, make sure not to move either the compass or the perpendicular direction indicator. Reliable alignment is *crucial* to the quality of your measurements!

3. Place the magnet upon its base, ensuring that its axis lies exactly along the perpendicular to the compass needle's original orientation. Start about 60 cm away from the compass needle and record the deflection angle $\theta$ and the distance $r$ from the center of the compass needle to the center of the magnet, as shown above.

4. Move the magnet closer to the compass at equal intervals (3 cm intervals should enable a measurable change in the deflection angle between readings). Take care to keep the magnet axis aligned along the perpendicular direction and to keep the compass *fixed* in place. At each location, record the deflection angle as in step 3. Stop when the magnet is 33 cm from the compass (this should leave you with 10 total measurements). We assume that the magnet is an ideal magnetic dipole, which is only valid if $r$ is much larger than the magnet length.

5. In your lab report draw a diagram of the magnetic field vectors at the location of the compass. Show the following vectors: $\vec{B}_m$ = the field due to the dipole magnet, $\vec{B}_{e,h}$ = the horizontal component of the earth's magnetic field, and $\vec{B}_r$ = the resultant magnetic field vector (the direction the compass needle will point). The deflection angle $\theta$ is the angle between $\vec{B}_{e,h}$ and $\vec{B}_r$. Use the geometric relationship between $\vec{B}_{e,h}$ and $\vec{B}_m$, along with Eq. (1), to write an equation involving $\theta$ and $r^n$. From this equation, involving the variables $r$ and $\theta$, what kind of graph could you plot that would be ***linear*** in nature? Remember, your goal is to find an experimental value for the exponent $n$. Plot this graph and see if your data give the expected linear behavior.

6. Use linear regression analysis to determine an experimental value for $n$ and its uncertainty. The field of a magnetic dipole is identical to the field produced by a small current loop. Consult your textbook to find a theoretical expression for the magnetic field as a function of the axial distance away from a *small* loop.

## Turn It In

1. E-mail your instructor and your lab partners a copy of your "data" sheets with the results of this laboratory. Don't forget to include the error analyses, regression output, and associated graphs needed to obtain your results.

2. Each student should hand in an independent Lab Report page, including a discussion of the results. Don't forget to sign your name at the top of the page!

**NOTE:** If your data, even when "properly" linearized still are not very linear you might choose to fit a linear subset of your data, rather than the entire data set. If so, make this decision clear e.g. in the figure caption, on the report form, etc. Feel free to discuss this with your instructor, e.g. multiple graphs, fit to various subsets, could make an effective presentation.

# Lab 9: The Magnetic Field of the Earth

## Overview

In this laboratory we will measure the magnetic field of Earth by utilizing a compass and the magnetic field from a loop of current.

## Learning Objectives

The main goals will be to understand vector addition, in the form of magnetic fields. Also, we will put into practice the form of magnetic fields from loops of current.

## Introduction

The theory accepted today for the origin of the Earth's magnetic field is the motion of the plasma (a mixture of electrons and positive ions) located in the hot core of the Earth. This plasma creates convection currents due to the rotational motion of the planet (the same spinning motion that induces the day-to-night cycle). The charge-carrying convection currents move on circular paths such that the associated angular momentum opposes that of the spinning Earth. This arises from the conservation of total angular momentum of the Earth, as an isolated system.

### Tangent Galvanometer

In the **tangent galvanometer**, a small magnet, in the form of a compass needle, is pivoted at the center of a circular coil of wire. The compass is free to rotate in a horizontal plane. When an electric current flows in the coil, a magnetic field is established perpendicular to the plane of the coil. This magnetic field is oriented so that it is perpendicular to Earth's magnetic field. The resultant magnetic field at the center of the coil is, then, the vector sum of two fields: the magnetic field due to the earth and the magnetic field due to the current in the coil. The compass needle will point in the direction of the resultant magnetic field.

## Procedure

1. On your lab report, diagram the horizontal magnetic field vectors at the location of the compass and derive an equation relating the horizontal component of Earth's field ($\vec{B}_{e,h}$), the coil field ($\vec{B}_c$), and the compass angle ($\theta$).
2. Measure the coil radius, $R$, and estimate its error. Record these values in your spreadsheet.
3. Let no current flow through the coil and locate the compass needle at the center, in a plane parallel with the ground. The needle is oriented in the direction of the Earth's magnetic field. ***Carefully align the coil so that the needle is parallel with the diameter of the coil***.
4. Connect the cables from the power supply to the galvanometer connections for $N = 5$ turns. **Using the voltage knob on the power supply**, increase the voltage until the current through

the coil is sufficient to cause the compass needle to deflect through an angle of about $10°$. Record the current and deflection angle of the compass needle in a spreadsheet. Repeat this procedure to achieve deflections up to $60°$ in increments of $10°$.
**NOTE: Limit the current to 1 A**. For $N = 5$, you may need to skip the $60°$ measurement.

5. Repeat step 4 for $N = 10$ turns and $N = 15$ turns.
6. Using your spreadsheet, calculate the average $B_c$ for each angle measurement. In other words, average the $B_c$ values for $N = 5$, $N = 10$, and $N = 15$ to obtain one $B_c$ value for each value of $\theta$. Now, you should have six "$y$" values and six "$x$" values. Plot $y$ vs. $x$ and fit the data with a linear regression. Using the slope of your regression, find $B_{e,h}$ and its uncertainty.
7. Using a dip needle, record the value of the "dip angle" of Earth's magnetic field at your lab table. Record this value in your spreadsheet and report it on your lab report form.
8. In your lab report, sketch a vector diagram including Earth's magnetic field at your lab table and the horizontal component of this field. Write an equation which relates these two values.
9. Calculate the value of $B_e$ you find from above, along with its uncertainty (ignore the uncertainty on the dip angle in this calculation) and report this value in your lab report.
10. As part of the interpretation of your results in your lab report, compare your experimental values with the expected values for Beaumont (Lat: 30.0858333 N, Long: 94.1016667 W), which can be found at http://www.ngdc.noaa.gov/geomag-web/#igrfwmm. You may wish to consider comparing the horizontal and total fields separately, as one quantity may involve additional systematic uncertainties.

## Turn It In

3. E-mail your instructor and your lab partners a copy of your "data" sheets with the results of this laboratory. Don't forget to include the error analyses, regression output, and associated graphs needed to obtain your results.

Each student should hand in an independent Lab Report page, including a discussion of the results. Don't forget to sign your name at the top of the page!

# Lab 10: Charge-to-Mass Ratio ($e/m$) of the Electron

## Introduction

The purpose of this experiment is to measure the ratio of the charge on an electron to its mass ($e/m$). Because the electric charge can be determined independently, e.g. by the Millikan oil drop experiment, this experiment can be regarded as a measurement of the mass of the electron.

Around the turn of the last century, the phenomena of electrical discharges in gases occupied the attention of many of the foremost researchers in physics. During their investigations, it was found that the negative charges transferred in such discharges were associated with what were called "cathode rays." Cathode rays were later shown to be beams of electrons.

Cathode rays are deflected by electric and magnetic fields. Based upon what was, at the time, a reasonable assumption, namely that cathode rays were beams of charged particles, the behavior of these "rays" in electric and magnetic fields could be predicted by classical electromagnetic theory. Pioneering experiments showed that the ratio of the electric charge divided by the mass for these particle beams was constant, independent of the source of the cathode rays. This fact lent strong support to the idea of the "corpuscular" nature of electric charge, that is, there exists a fundamental "quantum" of electric charge of which no fractional part could be observed. This idea was later confirmed by the famous Millikan oil drop experiment. In this sense, the early determinations of the ratio $e/m$ by methods similar to those used in this experiment rank among the classic experiments of modern physics.

## Apparatus

The apparatus used for this measurement, consists of a spherical glass beam tube, containing helium vapor at low pressure, placed between a pair of Helmholtz coils. When the coils carry an electric current, the magnetic field produced is very uniform throughout the region in which the electron beam moves. The magnitude of this magnetic field can be calculated from the current flowing through the coils and the size and shape of the coils.

Electrons, emitted by a heated cathode, are accelerated by the electric field between the cathode and the anode. Some of these electrons are projected through a hole in the anode. The electrons are moving in a direction perpendicular to the magnetic field, and so will move in a circular path. The radius of this circle is related to the electric potential difference that accelerates the electrons and the magnitude of the magnetic field.

The path of the electrons is made visible by the fluorescence of the helium vapor that was put in the tube for this purpose. A small fraction of the electrons in the beam suffer collisions with helium atoms. Some of these collisions lift the atoms to excited energy states from which they emit visible light upon returning to the ground state. Thus, a small fraction of the electron beam is sacrificed to make visible the path followed by the electrons that do not collide with any helium atoms.

The procedure for this experiment consists of successively adjusting the current flowing in the Helmholtz coils (thus, changing the magnetic field strength) and the accelerating voltage so that the radius of the electron beam path is unchanged.

## Theory

### Magnetic Field

To analyze your measurements and ultimately determine the value of the electron's charge-to-mass ratio ($e/m$), you need to know the strength of the magnetic field produced at the location of the electron beam by the circular Helmholtz coils. These coils are arranged in a special geometry that makes the field $\vec{B}$ at the center of the coils very uniform. The magnetic field from a Helmholtz coil is given by the formula

$$B = \left(\frac{4}{5}\right)^{3/2} \frac{\mu_0 N}{a} I,$$

where $N$ is the number of turns and $a$ represents the radius of the coil. For our coil, $N = 130$ and $a = 0.15$ m. So, from the current $I$ running through the Helmholtz coils you can determine the applied magnetic field $B_{\text{app}}$ that the electrons experience in the vacuum tube (note: in this case, Earth's magnetic field has a negligible effect on the total field; so we will treat it as zero).

### Charge-to-Mass Ratio ($e/m$) of the Electron

Because the magnetic force acts at right angles to the motion of a charged particle in a magnetic field, it does not speed up or slow down the particle, but rather causes it to move in a curved path. If the magnetic field is uniform, the curved path is a circular arc, the radius of which is determined by equating the magnetic force ($F_B = evB$) to the required centripetal force:

$$F_B = ma_c$$

$$evB = m\frac{v^2}{r}.$$

The electrons have been accelerated to a speed $v$ by a potential difference $\Delta V$ called the "anode voltage." Energy conservation gives

$$e\Delta V = \frac{1}{2}mv^2.$$

You can combine the equations in a couple different ways: (1) you could solve directly for $e/m$ in terms of $B$, $\Delta V$, and $r$; or (2) generate a relationship that gives $e/m$ as a slope of a graph (that is, $y = (e/m)x + b$ for some $y$ and $x$ that involve $B$, $\Delta V$, and $r$). By now, you know that the second method is often the more robust; so, this is the approach we will follow.

## Procedure

1. Using the equations listed above, derive an expression for $e/m$ in terms of $B$, $\Delta V$, and $r$.
2. Turn on the power to the heater, first. It should be set to 6 V and left in this position.
3. Turn on the power to the Helmholtz coil.
4. Adjust the current through the Helmholtz coil to 1 A.
5. Adjust the accelerating voltage until an electron path radius of about 4-5 cm is measured.
6. Record the current and accelerating voltage in your spreadsheet.
7. Repeat the procedure, incrementing the coil current by 0.1 A, each time, and adjusting the accelerating voltage to maintain the consistent radius. Continue until you have 7 trials (up to 1.6 A). Record each current and voltage measurement in your spreadsheet.
8. Using Excel, perform the necessary calculations to obtain the physical quantities needed for determining $e/m$. Generate a linear plot of the relevant quantities with a trend line.
9. Calculate a linear regression of your above results to extract an experimental value for $e/m$ and an associated uncertainty. Report and interpret this result in your lab report.

## Turn It In

1. E-mail your instructor and your lab partners a copy of your "data" sheets with the results of this laboratory. Don't forget to include the error analyses, regression output, and associated graphs needed to obtain your results.

2. Each student should hand in an independent Lab Report page, including a discussion of the results. Don't forget to sign your name at the top of the page!

# Lab 11: Atomic Spectroscopy and the Balmer Series

## Introduction

The purpose of this experiment is to observe characteristic spectral lines for hydrogen, calculate their wavelengths, and extract a value for the "Rydberg" constant.

In the 1800s, a popular topic was the study of gas discharge tubes: tubes containing gas vapor, e.g. from hydrogen, neon, or mercury. In contrast to the continuous spectra from incandescent sources (e.g. light bulbs), gas discharge tubes emit discrete spectra with only certain wavelengths of light present. These spectra are characteristic of the atoms or molecules of the particular gas and can be used to identify the elemental sources of light, e.g. from stars. Such analyses are generally referred to as "spectroscopy."

One of the most popular elements for spectral study is hydrogen, due to the relatively simple structure of its visible spectrum. At the end of the century, the Swiss physicist, J. J. Balmer, developed an empirical formula for the wavelengths of the spectral lines of hydrogen:

$$\frac{1}{\lambda} = R\left(\frac{1}{2^2} - \frac{1}{n^2}\right),$$

for $n = 3$, 4, 5, and 6. The four lines in the visible spectrum are part of what is called the "Balmer series." The constant $R$ is called the "Rydberg" constant, named for the Swedish physicist, Johannes Rydberg, who also studied atomic spectra. Its value has been determined to be $R = 1.097\times10^{-2}$ nm$^{-1}$.

The reason behind the spectral lines was not known in the 1800s. However, it gave tantalizing clues to the bizarre physics of the atom and nucleus, revealed in the next century in the development of quantum mechanics.

## Apparatus

The apparatus used for this measurement, consists of a gas tube with hydrogen, a diffraction grating, and a spectrometer. Light from the hydrogen gas will be diffracted through the grating that will separate the different wavelengths. The diffraction angles for these different "lines" will be measured by rotating the scope until the line is visible through it and reading off the angle from the spectrometer. The angular position data will be recorded in Excel for analysis.

## Procedure

1. Align the discharge tube so that the emitted light will pass into the spectrometer.
2. Insert a diffraction grating (e.g. 300 or 600 lines/mm) into the holder in the middle of the spectrometer. Adjust the grating, as necessary, so that it is perpendicular to the light coming from the discharge tube. Record the line spacing in an Excel spreadsheet and your lab report.
3. Position the viewing scope so that it is pointing straight at the gas tube. Turn on the gas tube. Adjust the tube location until a strong, clear central line is visible through the scope.
4. The light from the tube will be diffracted through the grating, scattering the various wavelengths at different angles. Whilst looking through the scope, slowly and gently rotate it clockwise until you see the first violet line. Record the diffraction angle in Excel. One tip: measure the deflection on each side of the dial. They should be close to the same value. If they are significantly different, you may want to adjust your setup, e.g. verify that the diffraction grating is perpendicular to the incident light.
5. Repeat step 4 for each of the visible spectral lines (If all works properly you should see a total of four: violet, blue, green, and red. You need only worry about their first-order diffractions). Then, repeat the process for the lines on the right side of the central maximum.
6. Turn off your gas tube.

## Data Analysis

1. In an Excel spreadsheet, you should have recorded the first-order diffraction angles $\theta$ and corresponding $n$ values for each of the different spectral lines. For each line, calculate the average diffraction angle.
2. Using your knowledge of diffraction, calculate a wavelength for each of the four spectral lines and record them in your lab report.
3. Linearize the Balmer formula (into the form $y = mx + b$). Based upon your linear form, create relevant spreadsheet columns and an appropriate plot to display your results (don't forget to include a trendline). Use a ***linear regression*** to extract the value and uncertainty for the Rydberg constant. Note: $n = 3$ to 6 in order of *decreasing* wavelength.
4. Compare your extracted value to the expected value. Be quantitative!

## Turn It In

1. E-mail your instructor and your lab partners a copy of your "data" sheets with the results of this laboratory. Don't forget to include the error analyses, regression output, and associated graphs needed to obtain your results.
2. Each student should hand in an independent Lab Report page, including a discussion of the results. Don't forget to sign your name at the top of the page!

# Tips on Experimental Errors and Data Analysis

### Data

1) *Record **raw** measurements on the spreadsheet*, not subsequent calculated values. In this way, you can more easily find mistakes if they occur. For example, record the measured length, width, and height of a cube rather than its calculated volume.
2) ***Use a spreadsheet to do your calculations***, rather than a hand-held calculator, so that errors can be fixed more readily. Be sure to label the spreadsheet adequately so that you understand it later.
3) Label all rows and columns (including units) on your spreadsheet.
4) Add comments to your spreadsheet, as necessary.
5) Do not round off your answers until the last step of the calculation. Rounding-off too soon can introduce undesirable errors. Your final calculated result should be expressed with the proper number of significant figures, based upon your experimental uncertainty.
6) Quote your results as (result) ± (uncertainty in this result, with units), e.g., "The measured acceleration was $2.21 \pm 0.14\ \mathrm{m/s^2}$."

### Graphs

Graphs of data and calculated quantities must **always** have:

- *a label on the x-axi*s giving the quantity plotted and the physical units
- *a label on the y-axi*s giving the quantity plotted and the physical units
- an appropriate *legend* clearly distinguishing between sets of data, if more than one series (a set of data *or* a best-fit line) is plotted on the same graph
- a *figure title* relevant to that experiment (not "John and Dave's Data").

When you perform a linear regression analysis, you will obtain values for the slope of the line and the intercept of the line from the regression. In Excel, this output will look like

| | *Coefficients* | *Standard Error* |
|---|---|---|
| Intercept | 0.711245 | 0.021334 |
| X Variable | 23.14976 | 1.188314 |

where:

0.711245 is the numerical value of the intercept of the straight line, and,

23.14976 is the numerical value of the slope of the straight line.

The uncertainty in the intercept is 0.021334, and, the uncertainty in the slope is 1.188314.

**To obtain a linear regression in Excel**

Open any spreadsheet with $(x, y)$ data. Under the **Data** tab, in the **Analysis** box, select **Data Analysis**, and then **Regression**. If the **Analysis** box does not appear, it means the data analysis tools are not yet loaded (notify your lab instructor). With the cursor in the box labeled 'Input Y Range', highlight the correct block for the dependent $(y)$ variable on your spreadsheet. Then move the cursor to the box labeled 'Input X Range', and do the same for the independent $(x)$ variable. Then click **OK** and look at your results.

**Some useful rules:**

1. **The reported uncertainty should have 2 significant digits only.**

   E.g. Excel calculates the standard deviation or "error" in a list of numbers to be $\sigma = 0.0123$ m. Reporting your error or uncertainty as

| | |
|---|---|
| $\sigma = 0.0123$ m | is WRONG (too many sig. figs. given) |
| $\sigma = 0.012$ m | is CORRECT |
| $\sigma = 0.01$ m | is WRONG (too few sig. figs) except for a measurement from a device for which the uncertainty is known to one digit |

2. **If none specified, then it is roughly indicated by the number of significant figures.**

   E.g. $g$ is given as 9.802 m/s$^2$, what is the uncertainty in that number? Roughly 0.001 m/s$^2$.

3. **The number of significant figures given must be consistent with the uncertainty.**

   E.g. we have made a measurement of $g$ and we know it only as well as $\pm 0.011$ m/s$^2$. Therefore, writing

| | |
|---|---|
| $g = 9.80234 \pm 0.011$ m/s$^2$ | is WRONG (too many sig. figs. given). |
| $g = 9.802 \pm 0.011$ m/s$^2$ | is CORRECT. |

**Experimental Uncertainties**

Suppose that a quantity $x$ is measured. The uncertainty of the measurement is denoted $\sigma$ (the Greek letter "sigma"). The result is expressed as:

$$x \pm \sigma$$

Other common terms:

- fractional error $= \frac{\sigma}{x}$
- percentage error $= \frac{\sigma}{x} \times 100\%$
- z-value $= \frac{|x_{\text{measured}} - x_{\text{expected}}|}{\sigma}$

**Example:** "I measured Mary's height with a meter stick to be $1.542 \pm 0.005$ m. The percentage error in the measurement is $0.005/1.542 \times 100\% = 0.3\%$."

If the difference between your measured value and the expected value exceeds $2\sigma$, i.e. $z > 2$, you should attempt to account for the reason.

## Types of Errors:

**1. Systematic error:** These are errors that affect all measurements in the same way or in some regular manner and are usually issues involving the **measuring instrument** or **details left out of the theory** guiding the analysis. Systematic error affects the **accuracy** of the measurement.

**2. Random Error:** If a measurement is repeated several times, slightly different values will be found, some higher than average and some lower. This type of uncertainty usually results from the **observer's judgment**. Random error affects the **precision** of the measurement.

There are several ways to estimate or determine the amount of random uncertainty.

1. Perform a linear regression with Excel
2. Take many measurements (5 – 10 or more) and calculate the standard deviation.
3. Take a few measurements, find the difference between the largest and smallest values, and set the uncertainty to half that difference.
4. Use the manufacturer-provided level of uncertainty, e.g. a scale labeled "$e = 0.01$ g," indicating that the uncertainty is 0.01 grams. If the uncertainty is not labeled on a device, assume it is half the smallest division, e.g. 0.0005 m on a ruler with 1 mm segments.

## Identifying Systematic Error:

Assessing the amount of systematic uncertainty is less clear than with random error. When performing experiments that have an expected result, the "z-value" can suggest the presence of systematics. Typical sources are:

1. Erroneous Calibration: Because the calibration is applied to every single data point in the same way, all the data are shifted too high or too low.
2. Simplifying Assumptions in the Theory: When performing exacting experiments, approximations can lead to theories that do not adequately describe real-world results, e.g. neglecting air-resistance to simplify equations of motion.

## Interpreting Results with Errors

When the same experiment is done many times, the repeated measurements will usually distribute themselves according to a Gaussian function (also known as a "bell-shaped curve"). Such a distribution can be characterized by its mean (or "average") and its "standard deviation" or $\sigma$, describing the width. Approximately 68% of repeated measurements will fall within one $\sigma$ on either side of the mean, approximately 95% will fall within $2\sigma$, and 99.7% within $3\sigma$.

If calculated correctly, the error obtained using the techniques described previously gives an estimate of the standard deviation of numerous repeated trials. You can compare your result to an expectation by calculating how many $\sigma$ the result differs from the expected result. The difference between the measured and expected values is the "z-value," expressed as

$$z = \frac{|\text{measured} - \text{expected}|}{\text{uncertainty}}$$

An example is the following:

**Measured value of charge on an electron:** $Q = 1.573 \pm 0.018 \times 10^{-19}$ Coulombs (C)

**Expected value:** $1.602 \times 10^{-19}$ C

$$z = (1.602 \times 10^{-19} - 1.573 \times 10^{-19})/(0.018 \times 10^{-19}) = 1.6$$

The result of one experiment does not "prove" anything. However, the custom in physics is to interpret an experimental result less than $2\sigma$ from expectation as "**consistent** with the expected value." For the example above, you can write a conclusion as follows:

"My experimental result of $1.573 \pm 0.018 \times 10^{-19}$ C is 1.6 standard deviations away from the expected value of $1.602 \times 10^{-19}$ C and is therefore consistent with the expected value."

Results more than $2\sigma$ away from the expected values may suggest more than just random uncertainties are involved in the experiment. If this happens, report your results and attempt to list possible factors that could cause systematics. Systematic errors often exist due to incorrectly calibrated equipment, incorrect assumptions, or poor experimental technique. If a result is more than $3\sigma$ from expectation is typically said to be "**inconsistent**" with the expected value.

## Propagation of Errors (Combination of Errors)

When measured quantities are added, multiplied, etc., then their uncertainties must be combined in some way to find the total uncertainty in the result. The full rules about how to do this are somewhat complex (see your instructor if you wish to know more). There are some simple rules and approximate methods shown below that can be used in this course. In the examples, we'll use $x$ and $y$ as the measured values with uncertainties $\sigma_x$ and $\sigma_y$, respectively.

**Simple Rules**

1. **Addition or subtraction:** The uncertainties add like in Pythagorean's Theorem.

$$z = x + y \text{ or } z = x - y \quad \rightarrow \sigma_z = \sqrt{\sigma_x^2 + \sigma_y^2}$$

2. **Multiplication or division:** The fractional uncertainties add like in Pythagorean's Theorem.

$$z = xy \text{ or } z = \frac{x}{y} \quad \rightarrow \frac{\sigma_z}{z} = \sqrt{\left(\frac{\sigma_x}{x}\right)^2 + \left(\frac{\sigma_y}{y}\right)^2}$$

3. **Exponent:** The fractional uncertainty gets multiplied by the exponent.

$$z = x^b \rightarrow \frac{\sigma_z}{z} = b\frac{\sigma_x}{x}$$

**Example:**
You want to know the area of a table top. You measure the lengths of the sides to be $l = 2.307 \pm 0.005$ m and $w = 1.34 \pm 0.01$ m (perhaps the edges in this direction are more rounded). The area is $A = lw = 3.09\ m^2$. The uncertainty would be

$$\frac{\sigma_A}{A} = \sqrt{\left(\frac{\sigma_l}{l}\right)^2 + \left(\frac{\sigma_w}{w}\right)^2} = \sqrt{\left(\frac{0.005}{2.307}\right)^2 + \left(\frac{0.01}{1.34}\right)^2} = 0.008 \rightarrow \sigma_A = (3.09\ \text{m}^2)(0.008) = 0.02\ \text{m}^2$$

**Significant Figures:**
To indicate the precision of a result, we write a number with as many digits as are "significant." The number of significant figures in a number is defined as follows:

1. The leftmost non-zero digit is the most significant digit.
2. If there is no decimal point, the rightmost non-zero digit is the least significant digit.
3. If there is a decimal point, the rightmost digit is the least significant digit, even if a 0.
4. All digits between the least and most significant, inclusive, are counted as significant.

E.g. The following numbers each have four significant digits:

1234, 123400, 123.4, 1001, 1000., 10.10, 0.00001010

- In recording the result of a measurement or calculation, **one and only one** doubtful digit is retained. E.g. measuring a length with a meter ruler where the smallest scale division is 1 mm. A typical measurement would read 58.6 mm, since you can judge the measurement to tenths of a mm but not to hundredths.

- In addition or subtraction, do not carry the result beyond the first column which contains a doubtful figure.

| | |
|---|---|
| 2807.5 | (tenths are the doubtful digit) |
| 0.065 | (thousandths are the doubtful digit) |
| + 83.79 | (hundredths are the doubtful digit) |
| 2891.355 | |

Round off at the end of the calculation. So, the sum is correctly expressed as 2891.4

Note that in many cases of subtraction, the final answer has fewer significant figures than either of the original numbers.

| | |
|---|---|
| 49563 | (5 sig. figs.) |
| -49532 | |
| 31 | (2 sig. figs.) |

- In multiplication or division, carry the result only to the same number of significant figures as there are in that quantity in the calculation which has the fewest significant figures.

  E.g. $10.51 \times 0.041 = 0.43091$

  Since the quantity with the fewest significant figures is 0.041 (2 sig. figs.) then the answer should only be given to **two** significant figures, i.e. as 0.43.

- Round off your estimates of errors to two significant digits.
- **When dealing with measured values**, where the uncertainty has been estimated or determined, **match the final answer's number of significant digits to the level of uncertainty, so the final digit in both the value and the error is in the same position with respect to the decimal.**

  E.g. if the uncertainty is 0.013 m/s and the value is measured to be 2.3457 m/s, the correct representation is $2.346 \pm 0.013$ m/s.