

4. Find the first few terms of the Laurent series for the following function about the origin.

$$\frac{1}{z(z-1)(z-2)} = \frac{1}{z} \left( \frac{-1}{z-1} + \frac{1}{z-2} \right) \quad (\text{by Ehrke method})$$

$$\frac{-1}{z-1} = + (1+(-z))^{-1} \approx 1 + (+1)(+z) + \frac{(-1)(-2)(+z)^2}{2!} + \frac{(-1)(-2)(-3)(+z)^3}{3!} + \dots$$

$$= 1 + z + z^2 + z^3 + \dots$$

$$\frac{-1}{z-2} = -\frac{1}{z} \left( 1 + \left(\frac{z}{2}\right) \right)^{-1} \approx -\frac{1}{z} \left[ 1 + (+1)\left(\frac{z}{2}\right) + \frac{(-1)(-2)\left(\frac{z}{2}\right)^2}{2!} + \dots \right]$$

$$= -\frac{1}{z} \left[ 1 + \frac{z}{2} + \frac{z^2}{4} + \frac{z^3}{8} + \dots \right]$$

$$\frac{1}{z} \left( \frac{1}{1-z} - \frac{1}{z-2} \right) = \left( \frac{1}{z} + 1 + z + z^2 + z^3 + \dots \right) - \left( \frac{1}{2z} + \frac{1}{4} + \frac{z}{8} + \frac{z^2}{16} + \frac{z^3}{32} + \dots \right)$$

$$= \frac{1}{2z} + \frac{3}{4} + \frac{7z}{8} + \frac{15z^2}{16} + \frac{31z^3}{32} + \dots$$