MATH 227: Discrete Mathematics

Name: _____

Midterm Exam, Part 2
Fall 2010

You have 50 minutes to complete this exam. Please show all work. Answers containing only minor mistakes may receive partial credit. If you have any questions feel free to ask. Please read the instructions carefully. You are welcome to use your calculators on this exam.

1. Part II

This section of the exam will be very similar to your homework and is graded similarly. Below are five statements to be proved. **You are only responsible for submitting proofs for 4 of the 6 statements.** Each proof is worth 10 points with 5 points coming from mathematical accuracy, 3 points coming from organization, and 2 points coming from quality.

(1) Prove in two different ways that $| 2^A | = 2^{|A|}$. That is, prove in two different ways that for a set, A with n elements (i.e. $| A | = n$), the number of elements in the power set of A is 2^n.

M: _____ Q: _____ O: _____

(2) Give a combinatorial proof that

$$\sum_{i=1}^{n} i\binom{n}{i} = n2^{n-1}.$$

M: _____ Q: _____ O: _____

(3) Consider the seven letter word `armeyer`. How many sequences of seven letters do not contain the words `eye` or `ram`. (Hint: Use the inclusion-exclusion principle and count the complement.)

M: _____ Q: _____ O: _____

(4) If a and b are integers, then we say that $a \equiv b \pmod{m}$ if $m \mid (a - b)$. Prove that equality modulo m is an equivalence relation and describe the distinct equivalence classes.

M: _____ Q: _____ O: _____

(5) Let Σ be the alphabet $\{a, b\}$ and define Σ^* to be the set of all words (of any length) formed from the alphabet Σ. (A word is a string of letters and has no set length unless specified. For example a, aaa, $aaba$, $aaabba$... are all words formed from Σ and therefore belong to Σ^*.)

(a) Is there a function mapping Σ onto Σ^*? If yes, give an example. If no, explain why.

(b) Is there a function mapping Σ^* onto Σ? If yes, give an example. If no, explain why.

(c) Is there a 1-1 function mapping Σ to Σ^*? If yes, give an example. If no, explain why.

(d) Is there a 1-1 function mapping Σ^* to Σ? If yes, give an example. If no, explain why.

M: _____ Q: _____ O: _____

(6) Consider the integer equation $x_1 + x_2 + x_3 = 10$.

1. How many solutions, (x_1, x_2, x_3) does this equation have if each of x_1, x_2, x_3 are required to be positive integers?

2. How many solutions, (x_1, x_2, x_3) does this equation have if each of x_1, x_2, x_3 are required to be non-negative integers?

3. How many solutions, (x_1, x_2, x_3) does this equation have if $x_1, x_2 \geq 3$ and $x_3 = 0$ or $x_3 = 1$?

M: _____ Q: _____ O: _____