

Calculus II

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Type I Infin

Intervals

Example

Example

Example

Type 2: Discontinuo

Integrands

Example 5

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Comparison re

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Absolute

Example 10

#### Calculus II

Improper Integration

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Fall 2008



#### Improper Integrals

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Example 1

Example

Example

Example

Type 2: Discontinuo

Integrands

Example 5

Example 7

Comparison Te

Example 8

Absolute

Example 10

#### Integrals can commit two types of impropriety:

- Type 1 The interval of integration may be infinite. This is technically illegal, because the formal definition of definite integral relies on partitions that are finite.
- Type 2 The integrand may be unbounded on the interval of integration. This is also illegal, because the integrand is not defined at an endpoint of the interval of integration.

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Type I Infini

Intervals

Example 2

Example

Example

Type 2:

Discontinuo Integrands

Example 5

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Comparison T

Example 8

Example

Absolute Comparisons

Example 1

For each of the following integrals, determine the type of impropriety that occurs.

(a)

$$\int_{1}^{\infty} \frac{dx}{x^2}$$

(b)

$$\int_{1}^{\infty} \frac{dx}{x}$$

(c)

$$\int_{-\infty}^{\infty} \frac{dx}{1+x^2}$$

(d)

$$\int_0^\infty e^{-x^2} dx$$

(e)

$$\int_0^1 \frac{dx}{x^2}$$

(f)

$$\int_0^1 \frac{ds}{\sqrt{s}}$$

(g)

$$\int_0^\infty \frac{dx}{\sqrt{x} + x^2}$$

It will pay to keep these integrals in the back of your mind. They illustrate most of what can go right-and wrong-with improper integration.

#### Type I: Infinite Intervals

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#### Type I Infini Intervals

Example 2

Example 3

Example 4

Type 2: Discontinuo

Integrands

Example 6

Example 7

Example 8

Absolute

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#### Improper Integrals of Type I

(a) If  $\int_a^t f(x) dx$  exists for every number  $t \ge a$ , then

$$\int_{a}^{\infty} f(x) \ dx = \lim_{t \to \infty} \int_{a}^{t} f(x) \ dx$$

provided this limit exists (as a finite number).

(b) If  $\int_t^b f(x) dx$  exists for every number  $t \le b$ , then

$$\int_{-\infty}^{b} f(x) \ dx = \lim_{t \to \infty} \int_{t}^{b} f(x) \ dx$$

provided this limit exists (as a finite number).

(c) If both of the above integrals are convergent (i.e. their limits exist), then we define

$$\int_{-\infty}^{\infty} f(x) \ dx = \int_{-\infty}^{a} f(x) \ dx + \int_{a}^{\infty} f(x) \ dx.$$

We note that any value x = a can be used to split the integral.



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Determine whether the integral

$$\int_{1}^{\infty} \frac{dx}{x^{2}}$$



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Example

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Type 2: Discontinu Integrands

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Example 8

Example

Absolute Compariso

Example 1

Determine whether the integral

$$\int_{1}^{\infty} \frac{dx}{x}$$

Determine whether the integral

$$\int_{-\infty}^{\infty} \frac{dx}{1+x^2}$$



# Type 2: Discontinuous Integrands

#### Improper Integrals of Type 2

(a) If f is continuous on [a,b) and is discontinuous at b, then

$$\int_{a}^{b} f(x) dx = \lim_{t \to b^{-}} \int_{a}^{t} f(x) dx$$

if this limit exists (as a finite number).

(b) If f is continuous on (a,b] and is discontinuous at a, then

$$\int_{a}^{b} f(x) dx = \lim_{t \to a^{+}} \int_{t}^{b} f(x) dx$$

if this limit exists (as a finite number).

(c) If f has a discontinuity at  $c \in (a,b)$ , and both  $\int_a^c f(x) dx$  and  $\int_c^b f(x) dx$  are convergent, then we define

$$\int_{a}^{b} f(x) dx = \int_{a}^{c} f(x) dx + \int_{c}^{a} f(x) dx$$

and has limits according to those types above.



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Type 2: Discontinuo Integrands

Integrands

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Absolute

Example 10

Determine whether the integral

$$\int_0^1 \frac{dx}{x^2}$$



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Example 1

Intervals

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Type 2:

Discontinuo Integrands

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Comparison i

Evample

Absolute Compariso

Example 10

Determine whether the integral

$$\int_0^1 \frac{dx}{\sqrt{x}}$$



Determine whether the integral

$$\int_0^\infty \frac{dx}{\sqrt{x} + x^2}$$



#### Comparison Test for Improper Integrals

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Example 2

Example 3

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Example 10

#### Comparison Test

Suppose that f and g are continuous functions with  $f(x) \ge g(x) \ge 0$  for  $x \ge a$ .

- (a) If  $\int_a^{\infty} f(x) dx$  is convergent, then  $\int_a^{\infty} g(x) dx$  is convergent.
- (b) If  $\int_a^\infty g(x) dx$  is divergent, then  $\int_a^\infty f(x) dx$  is divergent.

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Does

$$I = \int_{1}^{\infty} \frac{dx}{x+1}$$

converge or diverge? Why?

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Discontinu Integrands

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Example

Absolute Comparison

Example

Does

$$I = \int_3^\infty \frac{2x^2}{x^4 + 2x + \cos x} \, dx$$

converge or diverge? Why?



#### Absolute Comparison Test for Improper Integrals

#### Absolute Convergence

Let f and g be continuous functions such that, for all x > a,

$$0 \le |f(x)| \le g(x).$$

Suppose that  $\int_a^\infty g(x) dx$  converges. Then  $\int_a^\infty f(x) dx$  also converges, and

$$\left| \int_{a}^{\infty} f(x) \ dx \right| \le \int_{a}^{\infty} g(x) \ dx.$$

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Comparison T

Example 8

Example

Absolute Comparison

Example 10

Does the integral

$$I = \int_{1}^{\infty} \frac{\sin x}{x^2} \, dx$$

converge or diverge? Why?