## **Abilene Christian University**

Fall 2011

## MATH 361.01: Ordinary Differential Equations

MWF: 8:00-8:50 Foster Science Building 239

Email: john.ehrke@acu.edu Phone: 325.674.2162 Course Blog: blogs.acu.edu/1210 MATH36101 Dr. John Ehrke Assistant Professor of Mathematics Office: Foster Science Building 229

**Required Text(s):** The following text(s) are required for this course and may be purchased in the campus book store, or ordered online at the student's discretion. Please bring these text(s) everyday to class.

 Elementary Differential Equations and Boundary Value Problems, Boyce, William E. and DiPrima, Richard C., 9<sup>th</sup> edition, John Wiley & Sons, Inc., 2009.

**Course Material(s):** There is no specific calculator required for this course, but students will find it helpful to have a calculator available for homework, in-class work and exams. Exams are designed to be done without calculators, and students will have access to Maple in the computer labs for assigned homework.

**Course Description:** This course expands on the ideas of calculus introduced in the calculus sequence. The student successfully completing this course will be able to combine analytical, graphical, and numerical methods to model physical phenomena described by ordinary differential equations. The ACU course catalog describes the course as follows:

**MATH 361 Ordinary Differential Equations** (3-0-3), spring, population models, first order differential equations, systems of first order differential equations and equilibrium points; oscillations and second order equations; Laplace transforms. Prerequisites: MATH 186 and MATH 187.

This course will engage students in not only symbolic manipulation of various differential equations, but will heavily emphasize the role of current computing technology in obtaining qualitative as well as quantitative information.

"...Technical computing environments like *Maple*, *Mathematica*, and MATLAB are widely available and now used extensively by practicing engineers and scientists. This change in professional practice motivates a shift from the traditional concentration on manual symbolic methods to coverage also of qualitative and computer based methods that employ numerical computation and graphical visualization to develop greater conceptual understanding. A bonus of this more comprehensive approach is accessibility to a wider range of more realistic applications of differential equations." <sup>1</sup>



<sup>&</sup>lt;sup>1</sup> Differential Equations and Boundary Value Problems: Computing and Modeling, 4<sup>th</sup> edition, Edwards & Penney, pp. vii

"...This [course] is about how to predict the future. To do so, all we have is knowledge of how things are and an understanding of the rules that govern the changes that will occur. From calculus we know that change is measured by the derivative, and using the derivative to describe how a quantity changes is what the subject of differential equations is all about.

Turning the rules that govern the evolution of a quantity into a differential equation is called modeling, and in this [course] we study many models. Our goal is to use the differential equation to predict the future value of the quantity being modeled.

"There are three basic types of techniques for making these predictions. Analytical techniques involve finding formulas for the future values of the quantity. Qualitative techniques involve obtaining a rough sketch of the graph of the quantity as a function of time as well as a description of its long-term behavior. Numerical techniques involve doing arithmetic (or having a computer do arithmetic) that yield approximations of the future values of the quantity. We introduce and use all three approaches in this [course]." <sup>2</sup>

**Mission Statement:** This course supports ACU's mission statement of preparing students for Christian service and leadership throughout the world by providing students a foundational understanding of the mathematical principles such as problem solving and decision making, as well as exposing students to the role of mathematics in a Christian world view.

**Departmental Mission:** The mission of the Department of Mathematics is to educate students to be quantitative and analytical thinkers in preparation for Christian service and leadership throughout the world.

**Grading Components:** This course employs a points scale and the specific grading components and associated percentages are described below.

2250-2500	2000-2249	1751-1999	1500-1750	Below 1500
Α	В	С	D	F

**Exams (60%):** There will be two midterm exams, and a final exam for this semester. The midterm exams will be completed outside class time, while the final exam will take place in class from **8:00 – 9:45**, on Wednesday, December **14**. Each of these exams is announced on the course schedule in this syllabus. Each exam will cover approximately two units of content and are each worth 500 points.

<sup>&</sup>lt;sup>2</sup> Differential Equations, 2<sup>nd</sup> edition, Blanchard, DeVaney, and Hall, Boston University, Brooks—Cole, pp. vii-1.

<u>Modeling Project (20%)</u>: Students are expected to research a topic in which they are interested and produce a work of some significance by the end of the semester. Students will have a variety of final products to choose from, including papers, screencasts, poster sessions, in-class presentations, etc... The project component of your grade is worth 500 points or 20% of your course grade.

**Homework (20%):** Homework will be assigned regularly throughout the semester. In total, there are 8 homework sets with 62 questions worth 10 points each. You can earn a maximum of 500 points toward your homework grade, with percentages based on this total. Approximately 300 points (i.e. 30 questions) on the homework are considered straightforward applications of the lecture notes and should be answered correctly by the majority of students. This leaves 320 points from which you must earn 150 points of credit for an A. **Homework will not be accepted late. In the event of an excused absence all homework will be subject to the attendance-make up policy for the course.** Students should expect to use *Maple* on some questions. The math department computer lab hosts a site license version of the software available for student use.

**Course Competencies:** The course competencies, written in student performance terms, are detailed in the table below.

Competency	Measurement Instrument	Measurement Standard
The student will be able to construct mathematical models with aid of a computer algebra system and investigate their behavior analytically, numerically, and graphically.	Exams Homework Maple explorations Modeling project	<ol> <li>The student will be able to analyze mechanical systems with and without forced oscillations via analytic and graphical comparisons.</li> <li>The student will illustrate and compare how systems of differential equations arise naturally in scientific problems.</li> <li>The student will demonstrate that the same mathematical model can describe different physical systems and appreciate the unifying role mathematics plays in the investigation of natural phenomena. (i.e. mechanical-electrical analogy)</li> <li>The student will be able to construct first order systems and implement parameter studies for linear and nonlinear models like the Lotka-Volterra model.</li> </ol>
The student will be able to manipulate differential equations to obtain qualitative and quantitative information about solutions.	Exams Homework Maple explorations	<ol> <li>The student will be able to identify a first order ODE as linear, separable, or as one that requires a substitution method and apply the appropriate solution technique.</li> <li>Students will identify the characteristic roots of a homogeneous second order ODE and apply this method to obtain particular and general solutions of the related inhomogeneous problem.</li> <li>The student will be able to derive solution techniques for linear systems of ODE's via matrix and eigenvalue methods.</li> <li>The student will obtain solutions for ODE's via power series techniques, and knowledge of special functions.</li> </ol>

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The student will	Exams		
demonstrate a grasp of the geometric significance of	Homework Maple explorations	1.	The student will be able to describe the properties of slope fields and phase portraits for autonomous ODE's including stability and
the solution to an ODE.			equilibria.
		2.	The student will observe bifurcations in parameter studies and
			produce bifurcation plots and diagrams to justify conclusions about a model.
		3.	The student will observe the impact of an impulse in the graph of a solution to a forced system and draw conclusions about this impact based on the amplitude of the forcing term.
		4.	The student will examine the critical point behavior of linear systems and use this as a catalyst to provide a characterization of nonlinear systems locally.
		5.	The student will classify the slope fields for ODE's via eigenvalue methods as sinks, sources, centers and/or nodes.

**Attendance Policy:** Your regular attendance is both necessary and expected. You are responsible for all material covered while absent and will be expected to take regularly scheduled exams at their designated times except under extraordinary circumstances at the discretion of the instructor. You will be notified each time you are absent. Tardiness of more than 15 minutes is considered an absence and will be recorded as such. Should your number of absences exceed 25% of the scheduled course dates you can be dropped from the course at the instructor's discretion. Please make every effort to attend class prepared and ready to participate.

There are some situations for which a student absence will be excused. It is the student's responsibility to document such absences so they are removed from your record. Among the reasons absences are considered excused are the following:

- Participation in a university sponsored activity. If you miss class for a university sponsored activity, I must have seven (7) days advanced written notice and you must make arrangements to complete your assignments before leaving. The sponsor of the activity is responsible for giving you a signed form on time. If I do not have the appropriate form in advance, the absence will be considered unexcused. This is university policy.
- 2. Death or major illness in your immediate family. Immediate family includes parents, siblings, grandparents, spouse, child, and other as deemed appropriate by the instructor.
- 3. Illness of a dependent family member.
- 4. Injury or illness that is too severe or contagious for you to attend class.
  - a. Injury or illness of three or more consecutive days. For injury or illness that requires you to be absent from classes for three consecutive days, you should obtain a medical confirmation note from your medical provider. A health care professional may provide written confirmation only if he/she is involved in your medical care. The medical confirmation note must contain the date and time of the illness and medical professional's confirmation of needed absence.

- b. Injury or illness less than three consecutive class days. These will be evaluated on a per case basis at the discretion of the instructor. These must be brought to the instructor's attention through email or written note of explanation within a week of the last date of absence in order to be considered.
- c. An absence for any other non-acute illness or medical service does not constitute an excused absence.
- 5. Mandatory admission interviews for professional or graduate schools which cannot be rescheduled. Receiving advising from another department or registering for classes should never be done during class time and do not constitute an excused absence.

Should an excused absence cause you to miss a test, the instructor will provide an opportunity for the test to be taken at a time that works for all parties involved. If an absence is excused and you miss a daily assignment or quiz, I will replace the grade on the assignment with the average of the previous assignment and the next assignment. This will not require the student to turn in such assignments, but it should be noted that the student is responsible for all material covered during the absence.

**Homework Policy:** Homework sets will be assigned periodically throughout the semester. It is the expectation of the course that you will be working on homework every day. You should form the habit of doing the relevant problems between successive lectures and not try to do the whole set the night before they are due. Solutions will be available on the afternoon of the day they are due, **so late homework is not acceptable**. I encourage collaboration in this course, but I insist on honesty about it. If you do your homework in a group, be sure it works to your advantage rather than against you. **Good grades for homework you have not thought through will translate to poor grades on exams.** You must turn in your own write-ups of all problems, and, if you do collaborate or use outside resources, you should reference them on your solution sheet. Failure to do so constitutes an act of academic dishonesty.

**Make-up Policy:** After an exam has been graded and handed back in class, it will not be accepted for a grade under any circumstance. In the case of a university excused absence, it is the student's responsibility to make arrangements with the instructor regarding due dates prior to the absence. Exams cannot be made up if missed except under extraordinary circumstances at the discretion of the instructor. There will not be any work accepted for extra credit.

Academic Integrity Policy: The university policy regarding academic integrity is available online at <a href="http://www.acu.edu/campusoffices/provost">http://www.acu.edu/campusoffices/provost</a>. Students found guilty of an act of academic dishonesty will be subject to the following disciplinary actions in this course.

**Second Occurrence:** A second violation will result in your withdrawal from the course with a grade of F. A recommendation for suspension from the university will be made by the department.

**Electronic Devices Policy:** Please turn off all cell phones, beepers, pagers, alarms, .mp3 players, etc... unless such devices are being used for class purposes as indicated by your instructor. Headphones, listening to music, texting, and other uses of these devices not for class purposes are strictly prohibited during class. Frequent disruptions or failure to abide by this policy will be viewed as disruptive behavior and are subject to being dismissed from class and being counted absent. If the disruptions continue you will be dropped from the course.

**Disability Accommodations:** If you have a documented disability and wish to discuss academic

accommodations, please feel free to contact me. The ACU Student Disability Services Office (a part of Alpha Academic Services) facilitates disability accommodations in cooperation with instructors. In order to receive accommodations, you must be registered with Disability Services and you must complete a specific request for each class in which you need accommodations. Contact Disability Services at 674-2667 for further information or to set up an appointment.

**Office Hours:** Below is my schedule for the fall 2011 semester. The times marked "Office Hours" represent the times I will make myself available to work with you on homework, understanding lectures, or for any other questions you might have. Please take advantage of these opportunities. If you find that none of these times work for you, feel free to email me at john.ehrke@acu.edu or call me at 674-2162 to set up an alternate appointment. No appointment is needed if you attend regularly scheduled office hours. This schedule is posted on the front of my office door as well.

Fall 2011	Monday	Tuesday	Wednesday	Thursday	Friday
8:00 - 8:30	MATH 361.01 FSB 239		MATH 361.01 FSB 239		MATH 361.01 FSB 239
8:30 - 9:00	MATH 361.01 FSB 239		MATH 361.01 FSB 239		MATH 361.01 FSB 239
9:00 – 9:30	MATH 186.01 FSB 239		MATH 186.01 FSB 239		MATH 186.01 FSB 239
9:30 - 10:00	MATH 186.01 FSB 239	MATW 120.03 FSB 204	MATH 186.01 FSB 239	MATW 120.03 FSB 204	MATH 186.01 FSB 239
10:00 - 10:30	MATW 020.03 FSB 204	MATW 120.03 FSB 204	MATW 020.03 FSB 204	MATW 120.03 FSB 204	Office Hours
10:30 - 11:00	MATW 020.03 FSB 204	MATW 120.03 FSB 204	MATW 020.03 FSB 204	MATW 120.03 FSB 204	Office Hours

11:00 - 11:30							
11:30 - 12:00	Lunch Montings						
12:00 - 12:30			Lunch - Meetings				
12:30 - 1:00							
1:00 - 1:30					Office Hours		
1:30 - 2:00		MATW 120.04 FSB 204		MATW 120.04 FSB 204	Office Hours		
2:00 - 2:30	MATW 020.04 FSB 204	MATW 120.04 FSB 204	MATW 020.04 FSB 204	MATW 120.04 FSB 204	Office Hours		
2:30 - 3:00	MATW 020.04 FSB 204	MATW 120.04 FSB 204	MATW 020.04 FSB 204	MATW 120.04 FSB 204	Office Hours		
3:00 - 3:30	Office Hours	Office Hours	Office Hours	Office Hours			
3:30 - 4:00	Office Hours	Office Hours	Office Hours	Office Hours			

**Course Schedule:** A tentative course schedule for the semester is detailed in the table below.

Week 1 Week 2	Aug 29 Aug 31 Sep 2 Sep 5 Sep 7 Sep 9	Syllabus, Classifying Differential Equations (Chapter 1)Separable Equations (Sections 2.1-2.3)Method of Isoclines, Slope Fields, Introduction to Maple *not in textIntegrating Factor Methods (Section 2.1-2.3)Autonomous Equations (phase line, stability, bifurcation) (Section 2.5)Exact Differential Equations (Section 2.6)			
Week 3	Sep 12 Sep 14 Sep 16	Euler's Method (Section 2.7) *not included on midterm or homework Existence and uniqueness theorems (Section 2.8) Complex variables and the complex exponential			
Week 4	Sep 19 Sep 21 Sep 23	Linear system response, gain, phase lag *not in text Wronskian, general theory (Section 3.1-3.2) Spring mass system, simple harmonic motion (Section 3.3, 3.7)			
Week 5	Sep 26 Sep 28 Sep 30	Damped harmonic motion, three cases (Section 3.3, 3.4) Operator methods for exponential signals <i>*not in text</i> Alternative methods (Section 3.5, 3.6)			
Week 6	Oct 3 Oct 5 Oct 7	Damped forced vibrations (Section 3.8) Resonance (Section 3.8) Use of a known solution to find another <i>*not in text (optional)</i>			
	Midterm 1 Available 10/7-10/10 (Due by 5:00 PM 10/10)				

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Week 7	Oct 10 Oct 12 Oct 14	Review of Power Series (Section 5.1) Airy's Equation (Section 5.2) Legendre's Equation (Section 5.3)
Week 8	Oct 17 Oct 19 Oct 21	Euler's Equation (Section 5.4) Method of Frobenius (Section 5.5 – 5.6) Fall Break – no class
Week 9	Oct 24 Oct 26 Oct 28	Gauss' Hypergeometric Equation * <i>not in text (optional)</i> Bessel's Equation (Section 5.7) The Laplace Transform: Basic properties (Section 6.1)

Week 10	Oct 31 Nov 2 Nov 4	The Laplace Transform: Applications to ODEs (Section 6.2) Convolution Integral (Section 6.6) Abel's Mechanical Problem *not in text (optional)
Week 11	Nov 7 Nov 9 Nov 11	Step functions, discontinuous forcing (Section 6.3, 6.4) Impulses, Dirac Delta (Section 6.5) Step response, unit impulse response
		Midterm 2 Available 11/11 – 11/14 (Due by 5:00 PM 11/14)
Week 12	Nov 14 Nov 16 Nov 18	Basic theory of first order linear equations (Section 7.3 – 7.4) Eigenvalues, eigenvectors: Three Cases (Section 7.5) Eigenvalues, eigenvectors: Three Cases (Section 7.6, 7.8)
Week 13	Nov 21 Nov 23 Nov 25	Trace determinant plane, sketching autonomous systems (Section 9.1) Thanksgiving Break – no class Thanksgiving Break – no class
Week 14	Nov 28 Nov 30 Dec 2	Applications of Fundamental Matrices (Section 7.7) Applications of Fundamental Matrices (Section 7.9) Autonomous systems and stability (Section 9.2)
Week 15	Dec 5 Dec 7 Dec 9	Local linearization near equilibria (Section 9.3) Applications to population models (Section 9.4, 9.5) <i>*optional</i> Nonlinear Pendulum <i>*optional</i>
		Final Exam 8-9:45 AM, Wednesday December 14

**Modeling Project:** For the modeling project this semester you must not only decide on the topic you wish to research, but the product you will produce detailing your research. This semester there will be four options available for your final product.

**Option A: Screencast Presentation** Students choosing this option must create a professional looking 10-15 minute screencast detailing their research. Your screencast should focus on the evolution of your chosen topic from basics/history of the model, to computation of results, and ending with interpretation of the results. Screencast projects lend themselves to topics that are very graphically interesting and can be described using animations or graphs within Maple. Resources and tips for screencasting are located below in this syllabus. To develop a good sense for creating screencasts students should collect several screencasts from various sources to review and emulate.

**Option B: LaTeX Article** Students choosing this option should expect to write an 8-10 page research article. The paper must be written and compiled using the LaTeX document preparation system. LaTeX is the industry standard in creating high quality technical documents, and part of this project is becoming comfortable with expressing detailed mathematical information professionally and concisely. If you are unfamiliar with LaTeX, I can point you in the direction of several resources to help get you started. Your final paper should include several quality references and detail the findings of your research topic. To develop a good sense for writing high quality mathematical papers, you should be looking at research articles of similar quality and scope in peer-reviewed publications.

**Option C: Class Presentation** Students choosing this option must create an interactive presentation involving prepared slides and/or activities which focus on describing your findings in detail. Your audience will consist of students from the class and other faculty within the department. Your presentations should be between 15-20 minutes. To develop a good sense for presenting mathematical material you should find videos of lectures or other presentations made in a similar situation, consider what works in front of an audience, and what doesn't, and prepare a professional product.

Option D: Poster Session Students choosing this option must create a poster detailing their research findings. Poster sessions are common occurrences at undergraduate research festivals. A successful poster carefully crafts the research question, explains the

steps carried out in performing the research, and clearly explains the research outcome. To develop a good sense for creating poster presentations you should find examples of poster sessions given here on campus as well as from other universities. Students should be prepared to stand up before faculty and students in class and answer questions about their projects.

Projects with presentation requirements will be organized into groups and given the opportunity to present their findings during a departmental lunch meeting later in the semester. I will apprise you of the date(s) and time(s) for your presentations after the second midterm. Students may choose to work on a project alone or collaborate with a group of no larger than three students. If you intend to work in a group let me know within the first two weeks of the course. Group makeup cannot be changed thereafter without approval from the instructor.

To help get you started, a wonderful list of student differential equations projects can be found at the following resources. Within the first week or two of class, I will hand out an exhaustive list of topics for you to consider. A decision on your project topic and product are due during class following the first midterm. I will provide a form for you to fill out and turn in prior to that date.

Student Projects in Differential Equations: http://online.redwoods.cc.ca.us/instruct/darnold/deproj/index.htm

Case Studies in Differential Equations: <u>http://www.ibiblio.org/links/webhtml/DEindex.html</u>

Internet Differential Equations Activities: http://www.idea.wsu.edu/projects.php