MATH 120: Quantitative Reasoning Name: ____________________

Reflection Prompt 1

Fall 2011

Reasoning With Uncertainty

> *It is remarkable that a science which began with the consideration of games of chance should become the most important object of human knowledge... The most important questions of life are, for the most part, really only problems of probability... – Pierre Simon de Laplace (1812)*

1 Prompt

On a daily basis we are forced to make decisions that have far reaching consequences, but yet are based on incomplete information. Even when we try to acquire all the relevant information pertaining to a topic of interest rarely are we able to draw conclusions with 100% certainty. A doctor does not know exactly what is wrong with a patient even though he has at his disposal all manner of tests and equipment. A teacher does not know exactly what a student understands in spite of assignments designed to discover so. When we make decisions, we have to use the information we have.

To make a good decision, in math class and in life, a student cannot simply assume what the world is like and act according to those assumptions. They must consider multiple possible contingencies and their likelihood. Consider the following example.

> Many people consider it sensible to wear a seat belt when traveling in a car because, in an accident, wearing a seat belt reduces the risk of serious injury. However, consider a person that commits to assumptions and bases its decision only on those assumptions. If the person assumes they will not have an accident, they will not bother with the inconvenience of wearing a seat belt. If they assume they will have an accident, they will not go out. In neither case would a person wear a seat belt! A more intelligent person may wear a seat belt because the inconvenience of wearing a seat belt is far outweighed by the increased risk of injury or death in the case of an accident. They do not stay at home too worried about an accident to go out; the benefits of being mobile, even with the risk of an accident, outweigh the benefits of the extremely cautious approach of never going out. The decisions of whether to go out and whether to wear a seat belt depend on the likelihood of having an accident, how much a seat belt helps in an accident, the inconvenience of wearing a seat belt, and how important it is to go out. The various trade-offs may be different for different people. Some people do not wear seat belts, and some people do not go out because of the risk of accident.

Reasoning under uncertainty has been studied in the fields of probability theory and decision theory. Probability is the calculus of gambling. When a person makes decisions and uncertainties are involved about the outcomes of its action, it is gambling on the outcome. However, unlike a gambler at the casino, we cannot opt out and decide not to gamble; whatever we do - including doing nothing - involves uncertainty and risk.

Many of us learn probability as the theory of tossing coins and rolling dice. Although this may be a good way to present probability theory, probability is applicable to a much richer set of applications than coins and dice. In general, we want a calculus for belief that can be used for making decisions.

The view of probability as a measure of belief, as opposed to being a frequency, is known as Bayesian probability or subjective probability. The term subjective does not mean arbitrary, but rather it means "belonging to the subject." For example, suppose there are three students, Alice, Bob, and Chris, and one die that has been tossed. Suppose Alice observes that the outcome is a 6 and tells Bob that the outcome is even, but Chris knows nothing about the outcome. In this case, Alice has a probability of 1 that the outcome is a 6, Bob has a probability of 1/3 that it is a 6 (assuming Bob believes Alice and treats all of the even outcomes with equal probability), and Chris may have probability of 1/6 that the outcome is a 6. They all have different probabilities because they all have different knowledge. The probability is about the outcome of this particular toss of the die, not of some generic event of tossing dice. These students may have the same or different probabilities for the outcome of other coin tosses.

The alternative is the frequentist view, where the probabilities are long-run frequencies of repeatable events. The Bayesian view of probability is appropriate for Christians because a measure of belief in particular situations is what is needed to make decisions. Humans do not encounter generic events but have to make a decision based on uncertainty about the particular circumstances they face. Probability theory can be defined as the study of how knowledge affects belief. Belief in some proposition, A, can be measured in terms of a number between 0 and 1. The probability A is 0 means that A is believed to be definitely false (no new evidence will shift that belief), and a probability of 1 means that A is believed to be definitely true. Using 0 and 1 is purely a convention.

Adopting the belief view of probabilities does not mean that statistics are ignored. Statistics of what has happened in the past is knowledge that can be conditioned on and used to update belief. We are assuming that the uncertainty is *epistemological* - pertaining to an agent's knowledge of the world - rather than *ontological* - how the world is. We are assuming that an agent's knowledge of the truth of propositions is uncertain, not that there are degrees of truth. For example, if you are told that someone is very tall, you know they have some height; you only have vague knowledge about the actual value of their height.

2 Instructions

Our most recent unit, Probability and Counting, considers the mathematical foundations of *reasoning with uncertainty*–the ability to represent the world by making appropriate assumptions, and how to reason with such representations. Many of you found yourselves stretched by this material. How much more are we stretched by our walk as Christians when, even though we have a great knowledge of things, we are faced with having to explain to others the reason for our faith–the reason we believe, even in the face of uncertainty.

Please follow the instructions below when submitting your manuscript:

- Your manuscript should be double spaced with one inch margins, using a single-clear 12 point font. Please do not include any extra space between paragraphs.
- Please include information identifying the author and title of the manuscript on every page along with page numbers.
- Your manuscript should be between 500-700 words (this is approximately 2.5 to 3.5 pages following the instructions above). Include your word count at the end of the manuscript.

3 Response to the Prompt

In response to the video by Peter Donnelly, "How statistics and probability fool juries," and in response to this prompt, please respond to the following questions in your manuscript.

- Explore the distinction between being a Bayesian probabilist and a frequentist in terms of how a Christian views the world.
- Thinking about the history of the Churches of Christ and Christianity in general, can you identify any topics over which the distinction between epistemological and ontological viewpoints have been a source of dispute.
- How has your study of probability influenced your ability to think more logically or more critically? What are the consequences of a poor background in the mathematical foundations of reasoning with uncertainty?
- How has your study of probability and the video we watched in class changed your skepticism regarding statistics and probabilities aimed at consumers? In other words, do you feel more prepared to judge the validity of quantitative information presented in the media?