Abilene Christian University

Department of Mathematics

MATH 227.01: Discrete Mathematics

MWF: 1:00 – 1:50
Foster Science Building 205

Email: john.ehrke@acu.edu
Phone: 325.674.2162
Course Blog: blogs.acu.edu/1020_MATH22701

Dr. John Ehrke
Assistant Professor of Mathematics
Office: Foster Science Building 229

Required Text(s): The following text(s) are required for this course and may be purchased in the campus book store, or ordered online at the student's discretion. Please bring these text(s) everyday to class.

1. *Discrete Mathematics with Applications*, 3rd edition, Susanna S. Epp, Brooks – Cole, 2004.
2. *How to Prove It: A Structured Approach*, 2nd edition, Daniel J. Velleman, Cambridge University Press, 2006.

Course Material(s): A calculator is recommended for this course. At a minimum, students should have access to a calculator for use on tests, quizzes, and homework. Calculators will not be loaned out during class and cannot be exchanged between students during tests or quizzes. The recommended calculators are the TI-83, TI-84, or TI-89.

Course Description: This course introduces mathematical reasoning and logic and serves as the foundation for more abstract mathematics. The course is designed to rely heavily on reading and writing. The proof techniques, definitions, and notations introduced in this course are ubiquitous in mathematics. The ACU course catalog describes the course as follows:

> **MATH 227 Discrete Mathematics** (3-0-3), fall, spring, elementary set theory, logic, combinatorics, relations and applications. Prerequisite: CS 115 or MATH 185.

This course is primarily an introduction to classical proof techniques in mathematics. Many students get their first exposure to mathematical proofs in a high school course on geometry. Unfortunately, students in high school geometry are usually taught to think of a proof as a numbered list of statements and reasons, a view of proof that is too restrictive to be very useful. There is a parallel with computer science here that can be instructive. Early programming languages encouraged a similar restrictive view of computer programs as numbered lists of instructions. Now computer scientists have moved away from such languages and teach programming by using languages that encourage "structured programming". The discussion of proofs in this book is inspired by the belief that many of the considerations that have led computer scientists to embrace the structure approach to programming apply to proof-writing as well. You might say this book teaches, "**structured proving**". [1]

One of the major expectations of this course is that students will be able to not only understand mathematical proofs, but to communicate mathematical ideas in a formal manner. ... "the book probably is different from your other math books, because the course itself is different … there's a lot of subject matter to cover, but even more important than the content itself is developing logical thinking. That's one of the main skills you'll get out of this course, but it won't come without

[1] Velleman, Daniel J., *How to Prove It: A Structure Approach,* pp. xi-xiii.

lots of practice. We'll emphasize the kinds of logic that go into mathematical proofs or into program verification, but the training will also be useful in whatever you do, help you think through complicated situations."

… As the book goes on, more and more exercises ask you for proofs. We use the word "show" most commonly when a calculation is enough of an answer and "prove" to indicate that some reasoning is called for. "Prove" means give a convincing argument or discussion to show why the assertion is true." **What you write should be convincing to an instructor, to a fellow student, and to yourself the next day.** Proofs should include words and sentences, not just computations, so that the reader can follow your thought processes.

…Writing a good proof is a lot like writing a good computer program. Using words either too loosely or too extensively (when in doubt, just write) leads to a very bad computer program and a wrong or poor proof. All this takes practice and plenty of patience. [2]

Mission Statement: This course supports ACU's mission statement of preparing students for Christian service and leadership throughout the world by providing students a foundational understanding of the mathematical principles such as problem solving and decision making, as well as exposing students to the role of mathematics in a Christian world view.

Departmental Mission: The mission of the Department of Mathematics is to educated students to be quantitative and analytical thinkers in preparation for Christian service and leadership throughout the world.

Grading Components: A course grading rubric is included in this syllabus. The specific grading components outlined in that rubric are detailed below.

> **Exams:** There will be five exams given during the semester. Each of the exams is announced in the course schedule included in this syllabus. The structure of each exam is basically the same: there will be a section devoted to definitions and terminology, a section devoted to "showing" various calculations, and a section devoted to "proving" various statements.
>
> **Final Exam:** The final exam for this course is comprehensive and will be largely based on the five exams taken during the semester. The final exam is schedule for **8:00 – 9:45 AM, Thursday May 6**. The final exam cannot be given in advance of the scheduled time and will only be given after the scheduled date under extraordinary circumstances as the discretion of the instructor.
>
> **Homework Sets:** Homework will be assigned regularly throughout the semester. In total, there are thirteen homework sets planned. The material each homework set covers is detailed in the course schedule. Each homework assignment will be worth 20 points. The homework component of your grade will consist of your **ten** best homework assignments. Homework will be due the Monday following the date it is assigned. This ensures you will always have a weekend to work on a given assignment. It may be the case at some point(s) in the semester multiple homework assignments will be due on the same Monday.
>
> **LaTeX Writing Assignments:** This course will introduce you to the LaTeX document preparation system. More information on these assignments will be given throughout the semester. Each assignment is worth 20 points (same as the homework), and must be completed or else you risk losing points toward your final grade. The course blog contains various resources for installing LaTeX on your computers or for getting started on one of the math lab computers.

[2] Ross, Kenneth A. and Wright, Charles R., *Discrete Mathematics*, pp. xv-xvi.

Course Competencies: The course competencies, written in student performance terms, for this course are detailed in the table below.

Competency	Measurement Instrument	Measurement Standard
The student will be able to apply various techniques to analyze and prove mathematical statements in a variety of abstract settings including, but not limited to, sequences, sets, and functions.	Exams Homework Assigned Readings	1. The student will develop methods for counting large finite sets without exhaustively listing their elements as well as classifying sets which are countable, uncountable, and countably infinite (as an application to computability). 2. The student will apply logical and algorithmic methods of proof, such as contradiction, contraposition, and induction, to prove mathematical statements. 3. The student will apply partitions and relations to link elements of two different sets, or to compare and contrast members of the same set. 4. The student will use sequences to study repeated processes and verify conjectures about patterns governing the arrangement of terms in sequences. 5. The student will consider a wide variety of functions focusing on those defined on discrete sets as well as properties of functions including one-to-one and onto, existence of inverse function, and the interaction of composition of functions.
The student will demonstrate a grasp of proper notation and terminology in constructing proofs, counterexamples, and algorithms to test mathematical conjectures.	Exams Homework Assigned Readings	1. The student will articulate formal proofs which show competency in the use of proper mathematical notation and terminology. 2. The student will assimilate set notation and definitions to establish properties of sets through the use of proof and counterexample. 3. The student will compare and contrast the traditional mathematical number systems and representations with the definitions and terminologies used in computer representations of numbers and algorithms.
The student will be able to apply standard logical language and convention to mathematical statements.	Exams Homework Assigned Readings	1. The student will be able to construct truth tables for various compound statements. 2. The student will be able to prove a statement using the contrapositive. 3. The student will be able to analyze an argument for validity. 4. A student will be able to distinguish between constructive and non-constructive proof types. 5. A student will be able to form the inverse, converse, and contrapositive of a conditional statement. 6. A student will be able to write formal negations of statements that contain universal and existential quantifiers.

Attendance Policy: Your regular attendance is both necessary and expected. You are responsible for all material covered while absent and will be expected to take regularly scheduled exams at their designated times except under extraordinary circumstances at the discretion of the instructor. You will be notified each time you are absent. Tardiness of more than 15 minutes is considered an absence and will be recorded as such. Should your number of absences exceed 25% of the scheduled course dates you can be dropped from the course at the instructor's discretion. For this course, there are 43 class meetings scheduled, so this works out to a total of 10 absences for the semester. Please make every effort to attend class prepared and ready to participate.

Make Up Policy: Homework and LaTeX assignments will be due the first Monday after they are assigned by 3:00 PM. Homework not turned in before 3:00 PM can still be turned in for a penalty of 5 points (this is 25% of the assignment's worth), up until it is handed back in class, usually by Wednesday. **After an assignment has been graded and handed back in class, it will not be accepted for a grade under any circumstance.** In the case of an university excused absence, it is the student's responsibility to make arrangements with the instructor regarding due dates. **Exams cannot be made up if missed except under extraordinary circumstances at the discretion of the instructor.**

Academic Integrity Policy: The university policy regarding academic integrity is available online at http://www.acu.edu/campusoffices/provost. Students found guilty of an act of academic dishonesty will be subject to the following disciplinary actions in this course.

> **First Occurrence:** A first violation will result in no credit for that particular assignment (even if it is an exam). No makeup will be allowed. The appropriate campus office(s) will be notified of the incident, and a notice of the incident will accompany your university records.
>
> **Second Occurrence:** A second violation will result in your withdrawal from the course with a grade of F. A recommendation for suspension from the university will be made by the department.

Electronic Devices Policy: Please turn off all cell phones, beepers, pagers, alarms, .mp3 players, etc… unless such devices are being used for class purposes as indicated by your instructor. Headphones, listening to music, texting, and other uses of these devices not for class purposes are strictly prohibited during class. Frequent disruptions or failure to abide by this policy will be viewed as disruptive behavior and are subject to being dismissed from class and being counted absent. If the disruptions continue you will be dropped from the course.

Disability Accommodations: If you have a documented disability and wish to discuss academic accommodations, please feel free to contact me. The ACU Student Disability Services Office (a part of Alpha Academic Services) facilitates disability accommodations in cooperation with instructors. In order to receive accommodations, you must be registered with Disability Services and you must complete a specific request for each class in which you need accommodations. Contact Disability Services at 674-2667 for further information or to set up an appointment.

Course Schedule: A tentative course schedule for the semester is detailed in table below. Each row of the table represents a separate class meeting date, the lecture topic for that date, as well as reading(s) and suggested homework(s) from the text. All suggested homework, is only that, suggested, but will greatly increase your understanding of the material and serves as a source for potential exam questions. Graded homework assignments will be handed out in class and may include some problems from the suggested homework.

Lecture Topic	Reading(s)	Supplementary Reading(s)	Suggested Homework
Lecture 1: Syllabus, Truth Tables (negation, conjunction, disjunction), Logical Equivalences	Epps: pp. 1-17	Velleman: pp. 8-14	Section 1.1: #14, 16, 18, 23, 25, 26, 45, 47, 50, 52
Lecture 2: Truth tables (conditional, bi-conditional), Necessary and Sufficient Conditions, Only If	Epps: pp. 17-29, 94-95	Velleman: pp. 14-26	Section 1.2: #12, 13, 14, 29, 30, 31, 38, 40, 42
Lecture 3: Valid, Invalid Arguments, Universal and Existential Quantifier	Epps: pp. 29-43, 75-88	Velleman: pp. 26-34, 55-73	Section 1.3: #6, 8, 14, 18, 27, 28, 29, 41, 43
Lecture 4: Contrapositive, Converse, Inverse, Negation of Multiply Quantified Statements, Errors	Epps: pp. 88-94, 97-111	Velleman: pp. 34-54	Section 1.2: #20, 22, 23, 24, 25, 26, 27, 44, 45 Section 2.1: #14, 19 Section 2.2: #18, 20, 22, 24, 29, 31, 33, 35, 39, 42 Section 2.3: #41, 42, 43, 44 Section 2.4: #17, 18
Lecture 5: The Role of Logic in Proof	Epps: pp. 111-124	Velleman: pp. 73-83	
Test 1: Introduction to Logic			
Lecture 6: Number Theory I: Even, Odd, Rational, Irrational, Prime, Composite (Direct Proof)	Epps: pp. 125-141	Velleman: pp. 84-95	Section 3.1: #35, 36, 39, 43, 50, 52, 58, 59
Lecture 7: Number Theory II: Even, Odd, Rational, Irrational, Prime, Composite (Proof by Contradiction)	Epps: pp. 141-147, 171-175, 179-185	Velleman: pp. 95-107	Section 3.2: #27, 28, 29 Section 3.7: #19, 20, 21, 23, 26, 32
Lecture 8: Number Theory III: Divisibility, Quotient Remainder Theorem, (Proof by Cases)	Epps: pp. 148-156	Velleman: pp. 108-124	Section 3.3: #15, 17, 19, 21, 22, 23, 24, 33, 34, 41, 44, 45
Lecture 9: Number Theory IV: Divisibility, Division Algorithm, (Indirect Proof, Contraposition)	Epps: pp. 156-164, 175-178, 190-191	Velleman: pp. 124-146	Section 3.4: #26, 27, 28, 29, 30, 35-42 Section 3.6: #12, 13, 15, 23, 24, 25, 32
Lecture 10: Floor and Ceiling, *mod*, *div*, GCD, LCM (Constructive Proof Methods)	Epps: pp. 164-171	Velleman: pp. 146-162	Section 3.4: #1, 3, 5, 7, 43, 44, 45, 46, 47, 48 Section 3.5: #5, 13, 18, 19, 20, 28, 29
Lecture 11: Euclidean Algorithm, Finding the GCD, LCM, Tracing and Algorithm, (Introduction to Algorithms)	Epps: pp. 186-190, 192-198		Section 3.8: #8, 9, 13, 14, 17, 19, 23, 26, 28
Lecture 12: Extended Euclidean Algorithm, Writing the GCD as a Linear Combination	Epps: pp. 618-620		Section 10.4: #26, 27
Test 2: Introduction to Proof Techniques, Number Theory			
Lecture 13: Set Theory I: Subsets, Unions, Intersections, Partitions, Power Sets, Cartesian Products	Epps: pp. 255-269	Velleman: pp. 34-43	Section 5.1: #10, 14, 16, 20, 25, 26, 30
Lecture 15: Set Theory II: Element Arguments for Proving Subset Inclusion	Epps: pp. 269-282	Velleman: pp. 73-83	Section 5.2: #8, 10, 11, 12, 15, 16, 23, 25, 28
Lecture 15: Set Theory III: Proving Set Identities Algebraically, Counterexamples, Venn Diagrams	Epps: pp. 282-287	Velleman: pp. 163-171	Section 5.3: #5, 6, 11, 13, 14, 15, 17, 19, 20, 21, 23, 24, 25, 26, 32, 34
Lecture 16: Counting I: r—Permutations, Multiplication Rule, Permutations about a circle	Epps: pp. 297-320		Students will be given a large list of potential problems from the Epps and Ross & Wright texts. The exam for counting will come largely from these problems. This is done in an attempt to force students to pick the appropriate method for solving a given problem, rather than knowing the method ahead of time. This is generally what makes counting difficult for most students.
Lecture 17: Counting II: r—Combinations, Counting r—element subsets of a set	Epps: pp. 334-349		
Lecture 18: Counting III: Inclusion/Exclusion Principle	Epps: pp. 321-333		
Lecture 19: Counting IV: r—Combinations with repetition allowed, boxes and balls problem, counting iterations of a loop	Epps: pp. 349-355		
Lecture 20: Counting V: Counting unordered partitions, “paths” problem	Supplementary Notes		
Lecture 21: Counting VI: Binomial Theorem, Algebra of Combinations	Epps: pp. 356-369		Section 6.6: #6, 16, 17, 18 Section 6.7: #3, 5, 7, 11, 13, 15, 17, 18, 19, 24, 26, 28, 32, 34, 36
Test 3: Set Theory and Counting			
Lecture 22: Summation, Product, Factorial Notation	Epps: pp. 199-215		Section 4.1: #10, 11, 12, 14, 27, 32, 35, 52, 54, 58, 59, 60
Lecture 23: Weak Induction I: Proving Explicit Formulas for Sequences and Series	Epps: pp. 215-227	Velleman: pp. 260-267	Section 4.2: #2, 3, 6, 8, 10, 14, 15, 18, 19, 21, 23, 24, 27, 32

Lecture 24: Weak Induction II: Non-traditional inductive arguments, divisibility	Epps: pp. 227-235	Velleman: pp. 267-279	Section 4.3: #4, 8, 11, 12, 14, 15, 24, 25, 34 Section 5.2: #35, 36, 37
Lecture 25: Recursion, Explicit Formulas for Recursively Defined Sequences via Iteration	Epps: pp. 457-464, 475-483	Velleman: pp. 279-288	Section 8.1: #13, 14, 15, 36, 41, 42 Section 8.2: #3, 5, 6, 9, 10, 12, 14
Lecture 26: Famous Recursive Formulas: Fibonacci Sequence, Catalan Numbers, Stirling Numbers	Epps: pp. 464-475		Section 8.1: #26, 27, 31, 32, 44, 45, 48, 50, 51, 52
Lecture 27: Checking the correctness of a recursive formula via induction	Epps: pp. 483-487		Section 8.2: #43, 45, 46, 48, 49, 42, 53
Lecture 28: Strong Induction I: Unique Factorization Theorem, Well Ordering Principle	Epps: pp. 235-244	Velleman: pp. 288-300	Section 4.4: #7, 9, 11, 12, 14, 16
Lecture 29: Strong Induction II: Binary Representation of Integers, Number of Bits Needed to Represent and Integer in Binary Notation	Epps: pp. 235-244		Section 4.4: #27
Lecture 30: Loop Invariant Theorem, Application to Induction	Epps: pp. 244-254		Supplement from Ross & Wright
Test 4: Induction and Recursion			
Lecture 31: Relations on Sets, Functions as Relations, Directed Graphs	Epps: pp. 571-583	Velleman: pp. 171-180	Section 10.1: #5, 13, 14, 15, 16, 17, 18, 23, 25, 26, 29, 31
Lecture 32: Reflexive, Symmetric, Transitive, Equivalence Relations	Epps: pp. 584-594	Velleman: pp. 180-189, 202-213	Section 10.2: #1, 3, 6, 9, 10, 11, 14, 15, 18, 21, 23, 26, 30, 31, 37, 45, 47, 50
Lecture 33: Equivalence Classes, Partitions, Congruence modulo *n*	Epps: pp. 594-610	Velleman: pp. 213-225	Section 10.3: #2, 3, 5, 7, 8, 10, 14, 15, 18, 22, 23, 25, 27, 29, 31, 33, 35, 36, 40
Lecture 34: Modular arithmetic, modular equivalences, inverse modulo *n*	Epps: pp. 611-623		Section 10.4: #3, 6, 9, 11, 14, 15, 16, 31, 32
Lecture 35: Functions, Arrow Diagrams, Bizarre functions with computer science applications	Epps: pp. 389-402	Velleman: pp. 226-236	Section 7.1: #1, 6, 13, 14, 23, 24, 27, 31, 32, 34, 35, 36, 37, 38, 39, 40, 42, 44, 45
Lecture 36: One-to-one, Onto Functions, Inverses	Epps: pp. 402-419	Velleman: pp. 236-255	Section 7.2: #9, 10 (a,b), 11 (a,c), 21-27
Lecture 37: Pigeonhole Principle, Composition of Functions	Epps: pp. 420-443		Section 7.3: #14, 22, 24 Section 7.4: #5, 7, 16, 18, 19, 23, 26, 27, 29
Lecture 38: Cardinality of Sets, Countable, Uncountable, Countably Infinite	Epps: pp. 443-456	Velleman: pp. 306-322	Section 7.5: #2, 3, 8, 15, 16, 34, 35
Test 5: Relations and Functions			

Office Hours, Spring Schedule: Below is my schedule for the Spring 2010 semester. The times marked "Office Hours" represent the times I will make myself available to work with you on homework, understanding lectures, or for any other questions you might have. Please take advantage of these opportunities. If you find that none of these times work for you, feel free to email me at jee99a@acu.edu or call me at 674-2162 to set up an alternate appointment. No appointment is needed if you attend regularly scheduled office hours. This schedule is posted on the front of my office door as well.

<table>
<tr><th>Spring 2010</th><th>Monday</th><th>Tuesday</th><th>Wednesday</th><th>Thursday</th><th>Friday</th></tr>
<tr><td>8:00 – 8:30</td><td></td><td>MATH 120
FSB 241</td><td></td><td>MATH 120
FSB 241</td><td></td></tr>
<tr><td>8:30 – 9:00</td><td></td><td>MATH 120
FSB 241</td><td></td><td>MATH 120
FSB 241</td><td></td></tr>
<tr><td>9:00 – 9:30</td><td></td><td>MATH 120
FSB 241</td><td></td><td>MATH 120
FSB 241</td><td></td></tr>
<tr><td>9:30 – 10:00</td><td></td><td>Office Hours</td><td></td><td>Office Hours</td><td></td></tr>
<tr><td>10:00 – 10:30</td><td>MATH 361
FSB 239</td><td>Office Hours</td><td>MATH 361
FSB 239</td><td>Office Hours</td><td>MATH 361
FSB 239</td></tr>
<tr><td>10:30 – 11:00</td><td>MATH 361
FSB 239</td><td>Office Hours</td><td>MATH 361
FSB 239</td><td>Office Hours</td><td>MATH 361
FSB 239</td></tr>
<tr><td>11:00 – 11:30</td><td rowspan="4">Lunch</td><td rowspan="3">Lunch</td><td rowspan="4">Lunch</td><td rowspan="3">Lunch</td><td rowspan="4">Lunch</td></tr>
<tr><td>11:30 – 12:00</td></tr>
<tr><td>12:00 – 12:30</td></tr>
<tr><td>12:30 – 1:00</td><td>Office Hours</td><td>Office Hours</td></tr>
<tr><td>1:00 – 1:30</td><td>MATH 227
FSB 205</td><td>Office Hours</td><td>MATH 227
FSB 205</td><td>Office Hours</td><td>MATH 227
FSB 205</td></tr>
<tr><td>1:30 – 2:00</td><td>MATH 227
FSB 205</td><td>MATH 120
FSB 241</td><td>MATH 227
FSB 205</td><td>MATH 120
FSB 241</td><td>MATH 227
FSB 205</td></tr>
<tr><td>2:00 – 2:30</td><td>Office Hours</td><td>MATH 120
FSB 241</td><td>Office Hours</td><td>MATH 120
FSB 241</td><td></td></tr>
<tr><td>2:30 – 3:00</td><td>Office Hours</td><td>MATH 120
FSB 241</td><td>Office Hours</td><td>MATH 120
FSB 241</td><td></td></tr>
<tr><td>3:00 – 3:30</td><td></td><td></td><td></td><td></td><td></td></tr>
<tr><td>3:30 – 4:00</td><td></td><td></td><td></td><td></td><td></td></tr>
<tr><td>4:00 – 4:30</td><td></td><td></td><td></td><td></td><td></td></tr>
<tr><td>4:30 – 5:00</td><td></td><td></td><td></td><td></td><td></td></tr>
</table>