Investigations of the Number Derivative

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The Number Derivative

Theorem (Prime Factorization Theorem)

All positive integers $n \geq 2$ *can be written as* $n = \prod_{i=1}^{k} p_i^{x_i}$ *for distinct primes* p *and positive integers* x. *This factorization is unique up to order.*

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Examples and Domains

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$$\begin{aligned} \left(\frac{10}{9}\right)' &= (2 \cdot 5 \cdot 3^{-2})' \\ &= (2 \cdot 5 \cdot 3^{-2}) \cdot \left(\frac{1}{2} + \frac{1}{5} + \frac{-2}{3}\right) \\ &= \frac{5}{9} + \frac{2}{9} - \frac{20}{27} \\ &= \frac{1}{27} \end{aligned}$$

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   The Goldbach Conjecture states that every even number greater than 3 is the sum of two primes. So for two primes p and q, if there are solutions of type $n = pq$ for every $b > 1$, then the Goldbach Conjecture holds. (Ufranovski, 6)

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4. $n'' = 1, \quad$ (Twin Primes Conjecture)

   The Twin Primes Conjecture states that there are infinitely many prime pairs of the type p, $p + 2$. So if $n = 2p$, $n' = p + 2$, and thus if there are infinitely many solutions to $(2p)'' = 1$, then the Twin Primes Conjecture holds. (Ufranovski, 10)

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Since this matrix is diagonal the determinant of this matrix takes the same form as our factorization above.

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Consider a number n having only two prime factors, i.e. $n = p_1^{x_1} p_2^{x_2}$. We must show that our determinant gives us the same result when applying the product rule to this factorization, namely,

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We assume that the result holds for the above factorization, and we proceed by induction on k. Consider the case for $k+1$. We want to show that

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