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Ecological Modeling Project HW #2

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```
restart :
with(plots) :
with(DEtools) :
with(LinearAlgebra) :
```

a

> x1 := diff(x(t), t) = x(t)·(1 − 0.5·x(t) − 0.5·y(t))

$$x1 := \frac{d}{dt}\, x(t) = x(t)\,(1 - 0.5\,x(t) - 0.5\,y(t)) \quad (1.1)$$

> x2 := diff(y(t), t) = y(t)·(−0.25 + 0.5 x(t))

$$x2 := \frac{d}{dt}\, y(t) = y(t)\,(-0.25 + 0.5\,x(t)) \quad (1.2)$$

> solve([rhs((1.1)) = 0, rhs((1.2)) = 0], [x(t), y(t)])

$$[[x(t) = 0., y(t) = 0.], [x(t) = 2., y(t) = 0.], [x(t) = 0.5000000000, y(t) = 1.500000000]] \quad (1.3)$$

> The critical points of this system are (0, 0), (2, 0), (0.5, 1.5).

> $A := \begin{bmatrix} 1 & 0 \\ 0 & -0.25 \end{bmatrix}$;

$$A := \begin{bmatrix} 1 & 0 \\ 0 & -0.25 \end{bmatrix} \quad (1.4)$$

> Eigenvalues(A);

$$\begin{bmatrix} 1. + 0.\,I \\ -0.250000000000000000 + 0.\,I \end{bmatrix} \quad (1.5)$$

> At the critical point (0, 0), we have an unstable, saddle point.

> $B := \begin{bmatrix} -1 & -1 \\ 0 & .75 \end{bmatrix}$

$$B := \begin{bmatrix} -1 & -1 \\ 0 & 0.75 \end{bmatrix} \quad (1.6)$$

> Eigenvalues(B);

$$\begin{bmatrix} -1. + 0.\,I \\ 0.750000000000000000 + 0.\,I \end{bmatrix} \quad (1.7)$$

> At the critical point (2, 0), we have an unstable, saddle point.

> $C := \begin{bmatrix} -0.25 & -0.25 \\ 0.75 & 0 \end{bmatrix}$

$$C := \begin{bmatrix} -0.25 & -0.25 \\ 0.75 & 0 \end{bmatrix} \quad (1.8)$$

> *Eigenvalues*(*C*);

$$\begin{bmatrix} -0.124999999999999972 + 0.414578098794424922\,\mathrm{I} \\ -0.124999999999999972 - 0.414578098794424922\,\mathrm{I} \end{bmatrix} \quad \textbf{(1.9)}$$

> At the critical point (0.5, 1.5), we have a stable, spiral sink.

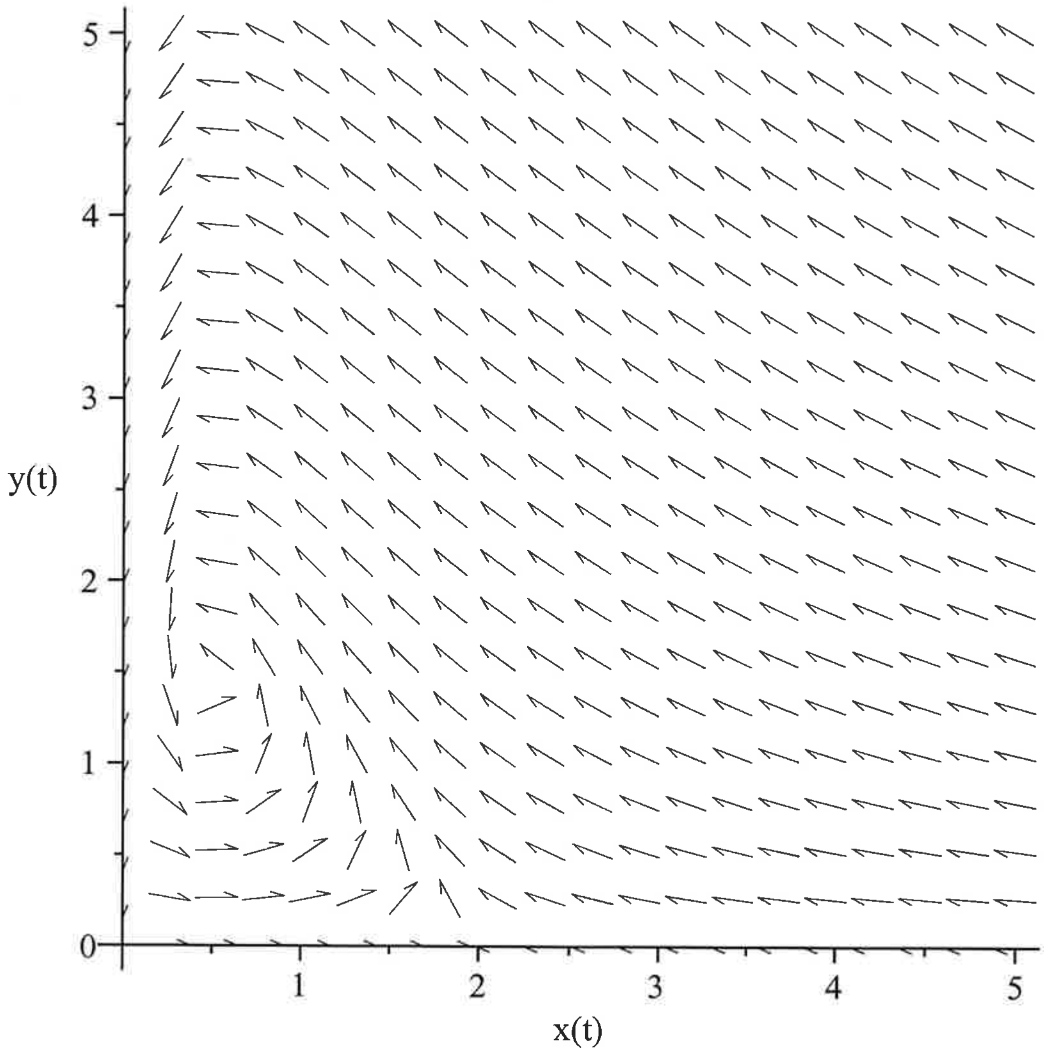
>

b

> *sys* := *x1*, *x2*

$$sys := \frac{d}{dt}\,x(t) = x(t)\,(1 - 0.5\,x(t) - 0.5\,y(t)),\ \frac{d}{dt}\,y(t) = y(t)\,(-0.25 + 0.5\,x(t)) \quad \textbf{(2.1)}$$

> *DEplot*({sys}, [*x*(*t*), *y*(*t*)], *t*=0..100, *x*=0..5, *y*=0..5, *scene*=[*x*(*t*), *y*(*t*)], *color*=*blue*);

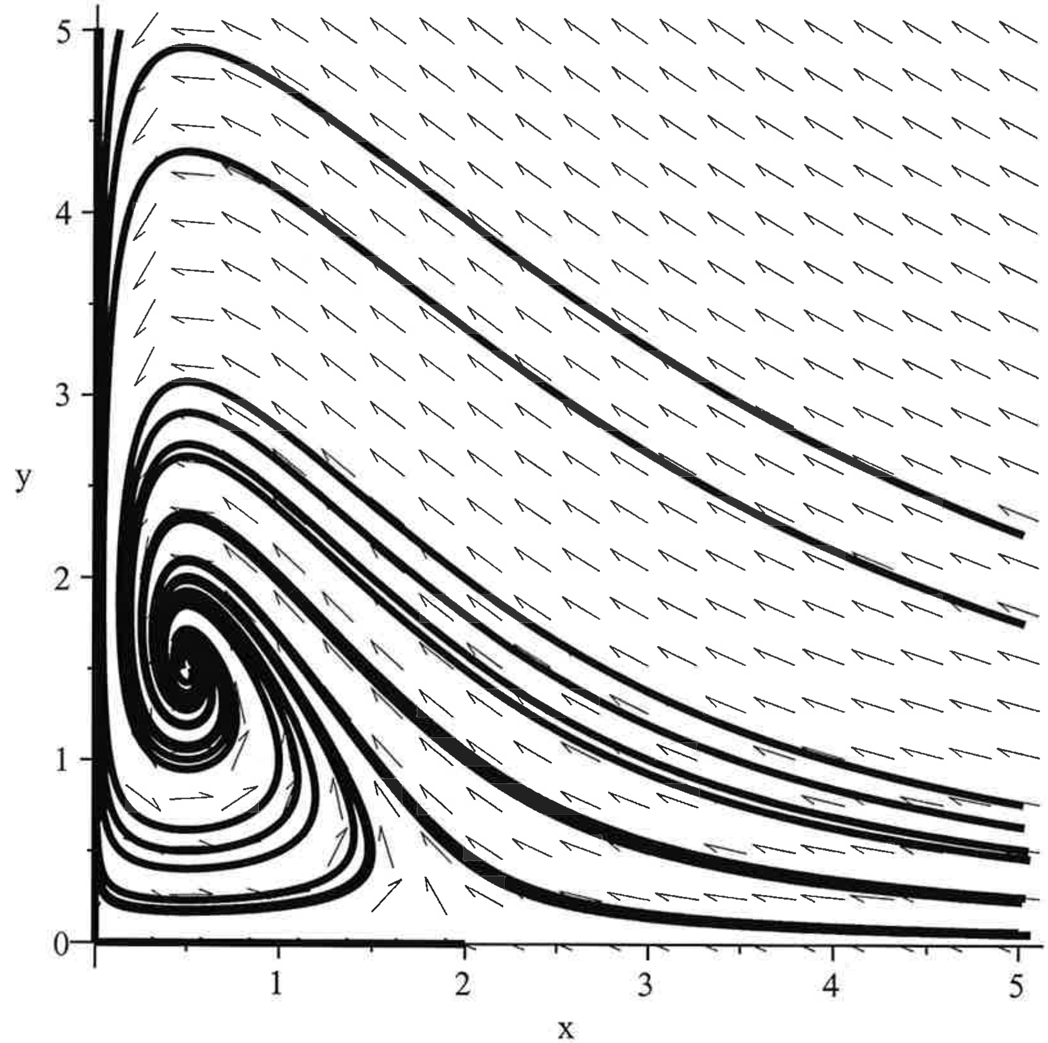


> The solutions seem to all spiral into the critical point at (0.5, 1.5).

c

> $ivals := \left\{ seq\left(seq\left(\left[x(0) = \frac{a}{2}, y(0) = \frac{b}{2} \right], a=0..4 \right), b=0..4 \right) \right\}:$

> *phaseportrait*({*sys*}, [$x(t)$, $y(t)$], t=-20 ..20, *ivals*, x=0 ..5, y=0 ..5, *stepsize* = 0.01, *linecolor* = *black*, *color* = *blue*);



> This model predicts that as t → ∞, all solutions, with positive initial conditions, approach the critical point at (0.5, 1.5).

d

For initial populations close to the critical point, the populations immediately enter this spiral sink towards the equilibrium populations. If one population is near 0, the the larger population decreases quickly until it is close enougth to the critical point to enter the sink. For initial populations both large, the predator population will increase while the prey decreases. Once the prey population nears zero, the the predator population decreases sharply until near zero, which allows the prey population to increase again. As it begins to increase, both populations get "sucked into" the sink and both head towards an equibrium state.

e

> *sys*

$$\frac{d}{dt}x(t) = x(t)\ (1 - 0.5\,x(t) - 0.5\,y(t)),\ \frac{d}{dt}y(t) = y(t)\ (-0.25 + 0.5\,x(t)) \qquad (5.1)$$

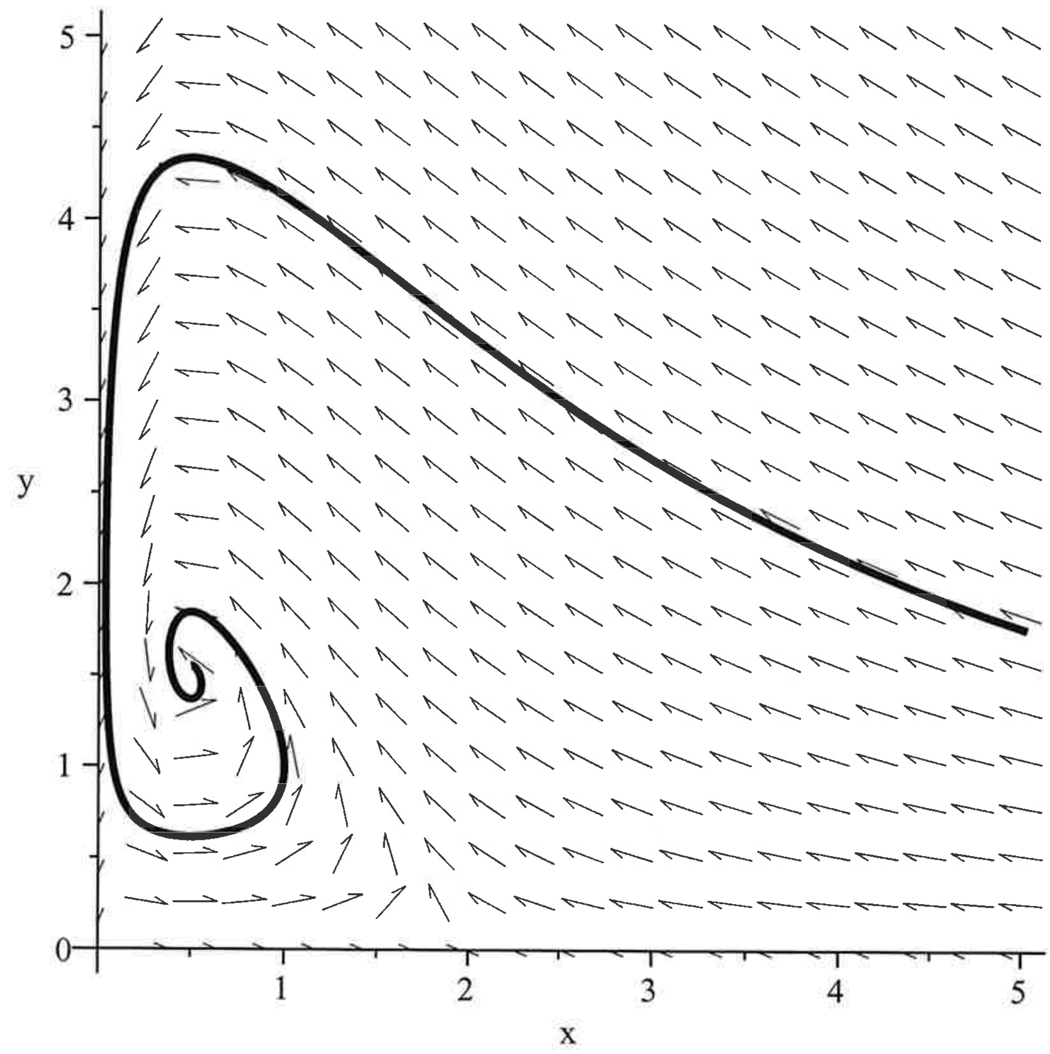
```
> dsolve[':-interactive']( {(5.1)} )
> ival := [x(0) = 1, y(0) = 1];
```

$$ival := [x(0) = 1, y(0) = 1] \quad (5.2)$$

```
> solv(1);
  solv(2);
  solv(3);
  solv(4);
  solv(5);
```

$$[t = 1., x(t) = 0.944598708844129110, y(t) = 1.27179653640938240]$$
$$[t = 2., x(t) = 0.815724882813373852, y(t) = 1.54080407646444706]$$
$$[t = 3., x(t) = 0.670943977663573277, y(t) = 1.73955708011590282]$$
$$[t = 4., x(t) = 0.548484453724478848, y(t) = 1.83513503840695113]$$
$$[t = 5., x(t) = 0.461639762512978780, y(t) = 1.83699634336366513] \quad (5.3)$$

```
> phaseportrait( {sys}, [x(t), y(t)], t=-20..20, [ival], x=0..5, y=0..5, stepsize=0.01, linecolor
     = black, color = blue)
```

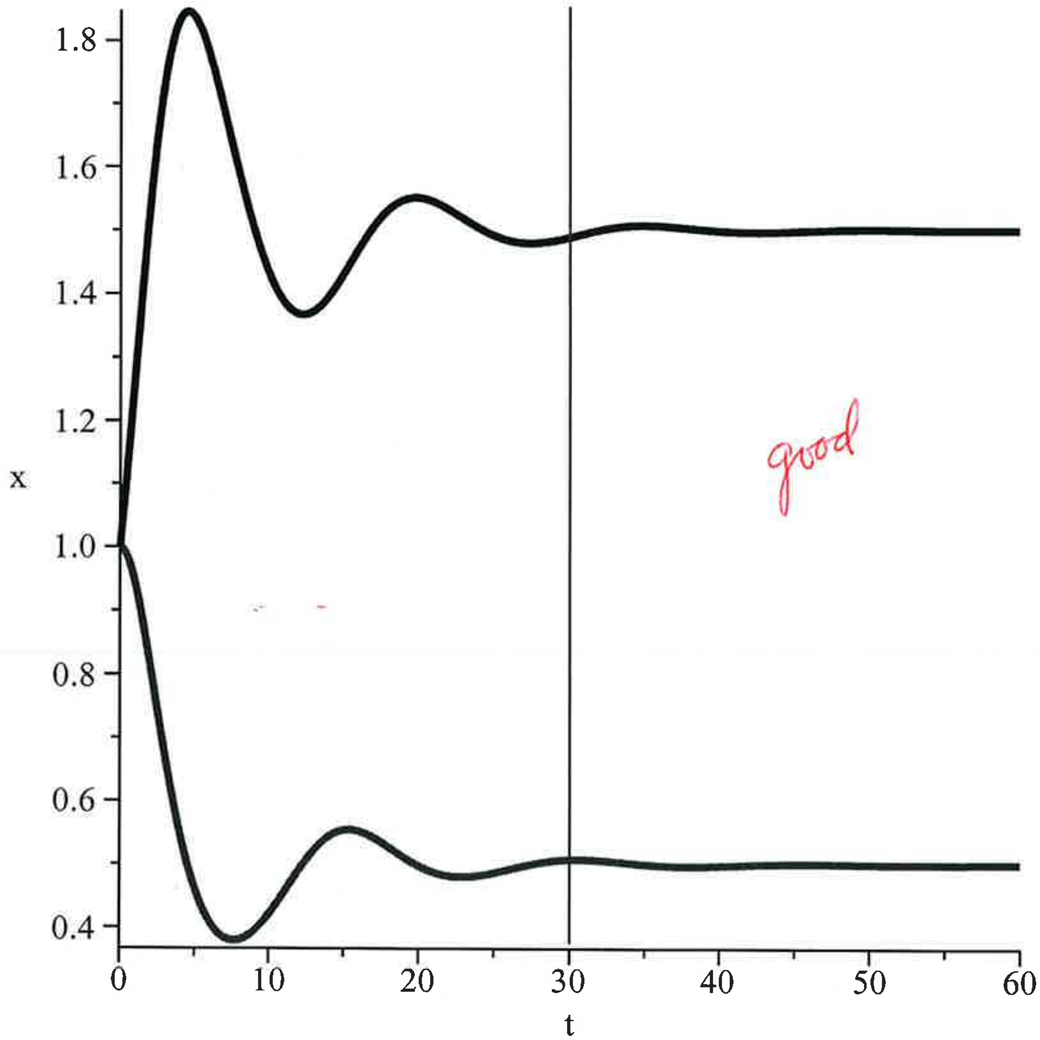



```
>
```

▼ f

```
> plot1 := DEplot( [x1, x2], [x, y], t=0..60, [ival], scene=[t, x], linecolor=red, stepsize=0.1,
```

```
    arrows = none) :
plot2 := DEplot([x1, x2], [x, y], t=0..60, [ival], scene=[t, y], linecolor=blue, stepsize=0.1,
    arrows = none) :
plot3 := plot([ [30, 0], [30, 2] ] ) :
display(plot1, plot2, plot3);
```



> It seems that, near t = 30, both populations are within 0.01 of their equilibrium values, 0.5 for the prey and 1.5 for the predators. Using the DE solver, it can be shown that this is the case.

```
> solv(30);
  solv(32);
  solv(34);
  solv(36);
  solv(38);
  solv(40);
```

$[t=30., x(t)=0.507703593116196838, y(t)=1.48930155513678876]$

$[t=32., x(t)=0.506417631215767660, y(t)=1.50048599926796711]$

$[t=34., x(t)=0.502069792862277264, y(t)=1.50699733737716324]$

$[t=36., x(t)=0.498307003536012150, y(t)=1.50703905382956949]$

$[t=38., x(t)=0.496971456239155452, y(t)=1.50315595404816360]$

$[t=40., x(t)=0.497835724861091956, y(t)=1.49905568212526164]$ **(6.1)**

>