DIFFERENTIAL EQUATIONS
- FINAL PROJECT -
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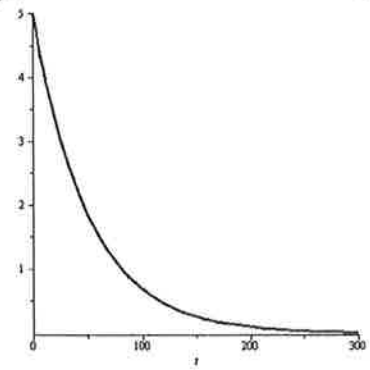


Mixing Problems

- Model the concentration of a substance dissolved in liquid
- Can model the inflow and outflow of the concentration
- The amount of the substance is Q(t)

Rate of change of Q(t) = rate of inflow of Q(t) - rate of outflow of Q(t)

Problem – Cascading Tanks

- More complicated problem using 2 tanks
- Inflow and outflow of all tanks is equal at 5 gal/min (volume of each tank is constant)
  Tank A = 100 gal
  Tank B = 200 gal

Initial Salt = Tank A and B have 20 lb.

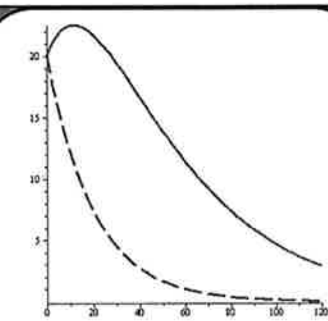
$$x'(t) = -\frac{1}{20}x(t),\ y'(t) = \frac{1}{20}x(t) - \frac{1}{40}y(t),\ x(0) = 20,\ y(0) = 20$$

Problem – One Tank

- Tank holds a constant amount of solution because inflow equals outflow

  <u>Example Problem</u>
  V = 500 Gal.
  Inflow = 10 gal/min x 0
  Outflow = 10 gal/min x Q(t)
  Initial Salt = 5 pounds

$$Q'(t) = -\frac{Q(t)10}{500} \qquad Q(0) = 5$$



Blue = Tank A   Red = Tank B

Source: www.publicintegrity.org

The Great Lakes - Modeling Equations

Superior: $x_s'(t) = r_s - x_s(t)\frac{15}{2900}$

Michigan: $x_m'(t) = r_m - x_m(t)\frac{38}{1180}$

Huron: $x_h'(t) = r_h + x_m t\frac{38}{1180} - x_s(t)\frac{15}{2900} - x_h(t)\frac{68}{850}$

Erie: $x_e'(t) = r_e + x_h(t)\frac{68}{850} - x_e(t)\frac{85}{160}$

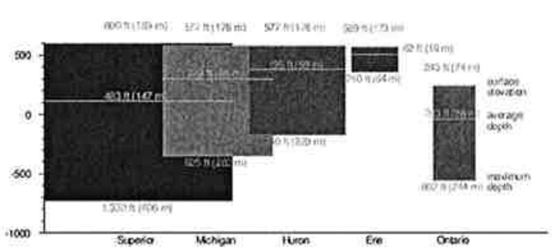
Ontario: $x_o'(t) = r_o + x_e(t)\frac{85}{160} - x_o(t)\frac{99}{393}$

The Great Lakes System

Nonpoint-Source Pollution and Point-Source Pollution

- Nonpoint-Source Pollution
  - Water pollution affecting a water body from diffuse sources, such as runoff
- Point-Source Pollution
  - Water pollution due to discharges into a body of water at a single location, such as from a factory

Great Lakes - Elevation



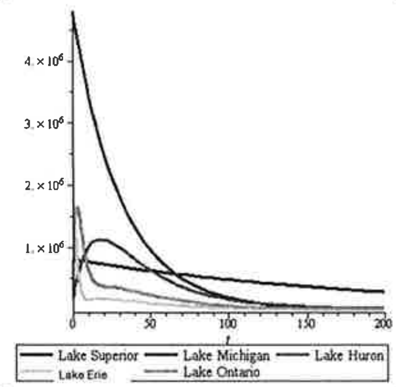
Major Pollutants

- MERCURY
- POLYCHLORINATED BIPHENYLS (PCBs)
- DIOXINS/FURANS
- HEXACHLOROBENZENE/BENZO(a)PYRENE (HCB/B(a)P)

Great Lakes - Current Pollution Levels

- Current Levels of Pollution in the Great Lakes from the year 2007 (all kinds of pollution)

Superior:	885,285 tons
Michigan:	4,793,877 tons
Huron:	163,790 tons
Erie:	2,988,934 tons
Ontario:	157,578 tons



Great Lakes - Yearly Pollution

- Amount of pollution released over the course of a year (all kinds of pollution)

Superior:	4,635 tons
Michigan:	48,423 tons
Huron:	7,445 tons
Erie:	114,959 tons
Ontario:	26,263 tons

Great Lakes – Only Input

- Here we will model the Great Lakes with the initial conditions set to zero.
- This lets us see the increase from no pollution to the effect of humans on the lakes

$$x_s'(t) = 4635 - x_s(t)\frac{15}{2900}$$
$$x_m'(t) = 48423 - x_m(t)\frac{38}{1180}$$
$$x_h'(t) = 7445 + x_m t\frac{38}{1180} + x_s(t)\frac{15}{2900} - x_h(t)\frac{68}{850}$$
$$x_e'(t) = 114959 + x_h(t)\frac{68}{850} - x_e(t)\frac{85}{160}$$
$$x_o'(t) = 26263 + x_e(t)\frac{85}{160} - x_o(t)\frac{99}{393}$$

Great Lakes – No Input

- We can model the Great Lakes with no input of pollution. In this case the variable r in the equations will be 0.

  With no pollution the current amount will go towards 0.
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  Superior:885,285 tons

Michigan:	4,793,877 tons
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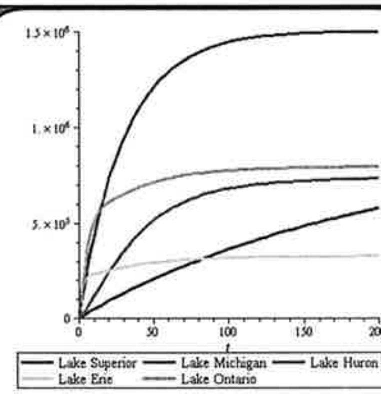

Great Lakes – Both IC and Input

- Now we want to see the effect of both Initial conditions in the lakes and the effect of pollution input

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$$x_m'(t) = 48423 - x_m(t)\frac{38}{1180}$$
$$x_h'(t) = 7445 + x_m t\frac{38}{1180} + x_s(t)\frac{15}{2900} - x_h(t)\frac{68}{850}$$
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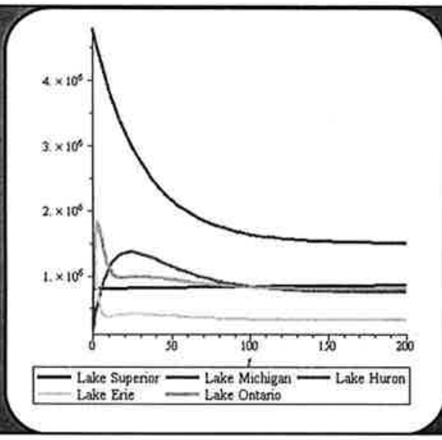
The Great Lakes

- From our data it is clear that Lake Michigan is a major contributor of pollutants
- We want to model what will happen if Lake Michigan does not contribute any pollutants to the system.
- It will only input clean water



How to Decrease?

- We want to be able to decrease the amount of pollution in all of the lakes
- It is difficult to focus on just one factory because:

Total Lake Erie:	114,959 tons
DTE Energy:	7,221 tons
Total Mercury:	1 ton

- The largest factory contributes less than 10% of the total
- Need to work to decrease all factory pollution by a small percentage