Pursuit Curves and Variable Scaling

Project Brief

Audience: Other students with your level of experience with topics in calculus and basic ODEs.
Due Date: This assignment is due 2 weeks after it is assigned.

When describing a model of pursuit, the pursuer follows a target (whose motion is known) by using a predetermined strategy. The typical strategy is to constantly aim at the target of pursuit. A typical example involving pursuit curves is outlined below.

Example 1. ***[Pursuit Curve Example]*** *A DEA lighthouse is looking for a drug trafficker in a local bay. One night the lighthouse beam shines directly on the boat of one of the drug traffickers. To evade the beam the drug trafficker will maintain an angle of* $45°$ *with the beam as it tracks the boat's getaway. What path does the boat follow while it evades direction?*

In addition to learning another useful model type, investigating *pursuit curves* often leads to a discussion of variable scaling in ODEs. Sometimes we scale variables in ODE to change an equation to a form where a solution is known while in the process reducing the number of symbolic coefficients appearing in the ODE. While it may not seem obvious at this point, the problem of identifying a path of pursuit typically involve solving a first order ODE $dy/dx = f(x, y)$ where $f(x, y)$ is a type of equation called a homogeneous equation.

Definition 1. ***[Homogeneous Equation]*** *An equation of the form*

$$\frac{dy}{dx} = f(x, y)$$

is called homogeneous (not to be confused) with linear homogeneous, if there exists a function g such that $f(x, y) = g(y/x)$. Saying $f(x, y)$ is homogeneous is equivalent to saying $f(kx, ky) = f(x, y)$ for some $k > 0$.

Some ideas your group may want to consider:

- Look at a very specific example which is modeled using a pursuit curve. Solve the model and discuss the role of variable scaling in your solution.
- Look at a very specific example which is modeled using a pursuit curve. Solve the model using a substitution which reduces the homogeneous equation to a known first order equation type.
- Discuss a variety of homogeneous equations that arise as solutions to models of pursuit curves and explain how different substitutions can reduce these equations to different types of solvable first order ODEs.
- Derive a solution model for a pursuit curve problem in which you desire dimensionless quantities. Explain the need for dimensionless variables and the substitutions needed to make this happen.

Any of the above ideas (or one of your own) would be great places to start researching this topic and putting together a small talk on a particular topic of interest to your group. If you have any questions please do not hesitate to stop by and see me. You are encouraged to take advantage of the Digital Learning Studio for help in putting together a screen cast. The Learning Studio blog is located at http://blogs.acu.edu/learningstudio/.