PHYS 221: Engineering Physics I

Laboratory Manual

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Acknowledgments

This laboratory manual was compiled and edited by Dr. James Drachenberg.

Lab 1. One-dimensional Motion

Overview

Two fundamental scientific and engineering skills are making observations and interpreting those observations in a logical way. While these are "scientific" skills, they are also fundamental exercises in creativity. Formulating an expectation, inferring a cause from observed effects, predicting future behavior based upon present conditions, and presenting results persuasively all involve artistic components. In describing and representing an object's motion, we introduce several quantities that are perhaps already quite familiar. We often denote the **position** of an object relative to some origin as a vector

$$\vec{r} = (x, y, z). \tag{1-1}$$

The **displacement** of an object during some interval of its motion can be written

$$\Delta\vec{r} = \vec{r}_2 - \vec{r}_1 = (\Delta x, \Delta y, \Delta z). \tag{1-2}$$

To characterize how position changes over some interval of time, we define an object's **average velocity** over the time interval

$$\vec{v}_{\text{av}} \equiv \frac{\Delta\vec{r}}{\Delta t} = \frac{\vec{r}_2 - \vec{r}_1}{t_2 - t_1}. \tag{1-3}$$

When an object's motion is not changing, we say it moves with a "constant velocity." A change to an object's motion is usually an indication something *interesting* has happened, e.g. an interaction has taken place (more on that, later)! If an object's velocity is changing over time, i.e. it is not constant, we may be interested in the velocity of an object at a given instant, what we call its **instantaneous velocity**, $\vec{v}$. A useful way to conceive the instantaneous velocity is to think of it as the average velocity over a vanishingly small interval of time. To use the language of calculus,

$$\vec{v} \equiv \lim_{\Delta t \to 0} \vec{v}_{\text{av}} = \lim_{\Delta t \to 0} \frac{\Delta\vec{r}}{\Delta t} = \frac{d\vec{r}}{dt}. \tag{1-4}$$

An effective way to present the motion of an object is by plotting the position as a function of time. In such a plot, the instantaneous velocity is given by the slope of the line tangent to the position vs. time curve. It follows that plots of velocity vs. time are effective for visualizing whether an object is in a state of constant or changing motion.

Learning Objectives

- Gain both a qualitative and quantitative understanding of one-dimensional motion
- Gain an introduction to the analysis of experimental data and techniques we will use throughout the course of the semester.

Exercise:

Part A – Experiment and Data Collection:

To begin, we're going to keep the experiment simple. Nevertheless, at all times, feel free to offer suggestions or ideas to keep it creative! We'll start by observing objects in (approximately) one-dimensional motion. For example, consider the auto traffic on EN 16th. Do you think the motion of the "typical" car is constant or changing? Do you think it complies with the speed limit? Think of some ways you could test this and, as importantly, present the information in a persuasive manner to someone outside this class. Discuss these questions briefly on your report form. For this first exercise, we will develop a method together; but don't assume the instructor has the "right answer" tucked away somewhere. Your ideas may be more fun!

As part of our experiment, let's track the object's position, call it x, as a function of time. Do this for at least 8 different positions. What origin will you use for your measurements? Describe how you translate the positions you observe into "real" distances, e.g. in meters? We often call this process "calibration." Describe how you measure the time intervals for these distances. **Remember:** While we will do this exercise together, your own ideas are welcome! Don't be afraid to think outside the box! Whatever you decide, store the pre-calibrated or "raw" data in a spreadsheet. We will use these to carry out the analysis in the next part of the exercise. For example, store the raw positions in a column of your spreadsheet with their respective times in another column, lining up the data for a given measurement on the same row.

Part B – Data Analysis:

Convert the raw data stored in your spreadsheet into a "calibrated" list of positions and their times. If you are unfamiliar with using a spreadsheet, it will be tempting to calculate things by hand and then type the "answer" into the spreadsheet. Resist this urge! Let the spreadsheet do the boring work for you. You can verify your spreadsheet calculations by hand, if needed. After you have a calibrated list of positions and times, use those data to make a position vs. time plot. Make sure to label the axes of your plot appropriately, including proper units.

Next, use your position and time data to calculate the object's velocity for each interval. This is not exactly the instantaneous velocity, but it is a good way to approximate it. In fact, as your time intervals get increasingly smaller, these values approach the true instantaneous velocities. Using your calculated values, make a velocity vs. time plot. Given how you calculate the velocity, what value do you think you should associate with the velocity over a given interval? Give a short justification for what you use on your report form.

Part C – Interpretation of the Data:

What story do your data tell? Let's answer the basic question of whether the motion of your object is constant or changing. How might you draw such a conclusion from your plots?

Discuss this with your lab partners and instructor and give a brief explanation on your lab report. Whatever your conclusion, give a *quantitative* argument for it! In other words, cite a quantitative measure to justify your conclusion.

Another important question to address is whether ***systematic uncertainties*** exist for your experiment? Usually the answer is "yes," and the crux of the matter becomes whether they are *relevant*? Briefly discuss possible systematic uncertainties and why they are relevant or irrelevant.

Turn It In

1. E-mail to your T/A, instructor, and lab partners a copy of your spreadsheet with the results of this laboratory. Don't forget to include any error analyses, regression output, associated graphs, etc., needed to obtain your results.
2. Each student should hand in an independent Lab Report page, including a discussion of the results. Don't forget to sign your name at the top of the page!

Lab 2. One-dimensional Acceleration

Overview

Studying motion in one dimension gives us the chance to learn about basic kinematics, focusing on the concepts before folding in the mathematical complications associated with higher dimensions. A particularly useful example of one-dimensional motion arises from an object in free-fall near Earth's surface. In the absence of effects such as air resistance, an object held above the ground and released from rest will move along a straight line, accelerating toward Earth's center at a constant rate. The acceleration is typically approximated to two significant digits as $g = 9.8\ \mathrm{m/s^2}$ in the direction of Earth's center.

Learning Objectives

- Gain both a qualitative and quantitative understanding of one-dimensional motion
- Gain an introduction to the analysis of experimental data and techniques we will use throughout the course of the semester.

Exercise:

Part A – Experiment and Data Collection:

Observe an object in "free fall" (well, as close as you can get to free fall in the lab, anyway). With your lab group, select an appropriate object to drop (**note:** not every object is conducive to "free fall"). Then, develop a robust method to measure the vertical position as a function of time. Some assorted tools will be provided but feel free to brainstorm about what else might work. Think outside the box! If you think you've got a good tool that would work, feel free to bring your own to lab. Three pieces of advice:

1. Complicated ≠ better
2. Be creative
3. Have fun with it!

Take a practice run or two to hone your method. Once you have practiced and are satisfied with your method, run it **at least three times**. For each run, record the (vertical) position and time information for at least 15 different instances. Depending on your method, it might be advantageous to measure the same 15 positions or the same 15 time-intervals for each of the three runs; but I'll leave it to you to decide. Record and organize your data in a spreadsheet. On your lab report, briefly describe your setup, both in words and with a sketch.

Part B – Data Analysis:

Combine all the data from your runs to create a spreadsheet plot of the object's vertical position as a function of time. A few suggestions (by no means an exhaustive list):

1. If you have measured different vertical positions at different time intervals for each of your data runs, plot every point on the same graph at its respective time interval.
2. If you have measured the time intervals for the same 15 vertical positions in each run, plot the 15 positions at their *average* time interval for the three runs.
3. If you have measured the vertical positions for the same 15 time-intervals in each run, plot the 15 *average* positions at their time intervals.

Plot a trendline through the data points on your graph in the spreadsheet. Think about how you expect the data to behave and choose an appropriate functional form for the trendline. Make sure to display the equation on your graph.

Using the position and time data, calculate and plot two more graphs: velocity vs. time and acceleration vs. time. Just as you did with the previous plots, include trendlines for these graphs using appropriate functional forms based on your expectation.

Finally, turn your data into the best estimate you can make for the acceleration of your object in "free fall." There are many "correct" ways to do this, so discuss it with your lab partners to determine what you think is "best." A couple of options to consider would be a linear regression or an average of several calculated values. Don't forget to include with your answer a sensible estimation of your experimental uncertainty! You may calculate and report this answer on your spreadsheet but please also report it with appropriate units and significant figures on your lab report form.

Part C – Interpretation of the Data:

What story do your data tell? Are your results, e.g. your plots and your free-fall acceleration, consistent with your expectation? Be *quantitative*! Cite a measure to justify your conclusion. What ***systematic uncertainties*** could exist for your analysis? Are any of them relevant? Again, justify your statement by citing a quantitative measure.

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Lab 3. Momentum

Overview

One difficult aspect to solving a new problem is understanding where to begin. Often, a good approach is to start with something you know with certainty. For this and many other reasons, among the most powerful tools in science are what we call "conserved" quantities, i.e. those that cannot be created or destroyed. For any system, conserved quantities cannot change unless there is input or output across the system boundary. In the absence of input and output, you can always be certain that a conserved quantity remains unchanged.

You have come across many quantities describing objects in motion, e.g. position, velocity, and inertia. Unsurprisingly it is useful to define a *conserved* quantity that describes matter in motion. We find this in the product of an object's inertia and velocity, i.e. its "momentum":

$$\vec{p} = m\vec{v}. \tag{3-1}$$

Emmy Noether demonstrated that momentum conservation flows naturally from the idea that simply changing your location in space doesn't change the laws of physics. This notion of "translational symmetry" is a comforting one [what would study abroad be like?!]; but, to our best understanding, the universe is not *a priori* obligated so to behave! The validity of our assumption hinges entirely upon its agreement with experiment.

Let's keep it simple. Consider an isolated system of two particles colliding in one dimension:

$$\Delta p_x = \Delta p_{1x} + \Delta p_{2x} = 0. \tag{3-2}$$

If the inertias of the objects remain unchanged, we can quickly rephrase the statement as

$$m_1 \Delta v_{1x} + m_2 \Delta v_{2x} = 0 \tag{3-3}$$

$$\rightarrow \Delta v_{2x} = -\frac{m_1}{m_2} \Delta v_{1x}. \tag{3-4}$$

In other words, if our assumption (momentum is conserved) is true, we should see a linear relationship between the changes in velocity of the two objects. The question is…do we…?

Learning Objectives

- Gain both a qualitative and quantitative understanding of momentum
- Gain an introduction to formulating a hypothesis analytically and testing it quantitatively

Exercise:

Part A – Experiment and Data Collection:

Set up an (approximately) isolated system of two objects that can collide in one dimension. You may use anything upon which you can lay your hands or bring with you to the lab. Bring your two objects into collision and measure the change in their velocities before and after the collision. You may do this in whatever way you see fit! Discuss it with your lab partners before deciding upon your favorite method. A few options you could consider:

1. Use meter sticks to track initial/final positions and distances traveled
2. Use slow-motion video or stopwatches to record timing
3. Use motion sensors to calculate and track velocities over the duration of the collision

Note: *This list is not exhaustive, and you are encouraged to think outside the box*! Practice your method several times or try a few different methods until you are satisfied. Once you have settled on a method, run it **at least five times** to observe five different pairs of Δv. For each run, record the raw data in a spreadsheet. For example, if you decide to calculate the velocities by measuring the distances traveled and time intervals for each object before and after collision, record the raw distances and timing, not just the final, calculated velocities. On your lab report, briefly describe your method, using both words and a sketch.

Part B – Data Analysis:

Create a spreadsheet graph of one object's change in velocity as a function of the other object's change in velocity. Plot a trendline through the data points on your graph in the spreadsheet. Consider how the data should behave if momentum is conserved and choose an appropriate functional form for the trendline. Display the trendline equation on your graph.

Calculate a linear regression for the data you have plotted (if you are using Excel, this is easily done with the Analysis Tools package). Appropriately report the slope and intercept, including their uncertainties, and the R^2 from your regression on your report form.

Part C – Interpretation of the Data:

Are your results consistent with your hypothesis that momentum is conserved? Justify your conclusion quantitatively. Do any ***systematic uncertainties*** complicate your ability to draw a conclusion? Justify your statement by citing a quantitative measure.

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Lab 4. Galilean Relativity

Overview

Some things in nature will be observed the same by everyone, regardless their frame of reference. Others will depend entirely upon the observer's reference frame. For example, the speed of a car barreling down the highway will appear very different to someone standing on the roadside compared to what is observed by a motorcyclist riding on the highway alongside the car. If the observed speeds are different, it follows that the observed kinetic energy of the car must also be different. However, the amount of gasoline consumed by the car will be measured the same by both observers. The *change* in energy is known as an "invariant" quantity, i.e. one that does not depend upon an observer's frame of reference.

It is often easiest to work from "inertial" reference frames, i.e. those that move at constant velocities relative to each other. One of the main reasons is the laws of physics are the same in every inertial reference frame. An example of a non-inertial frame is a truck accelerating after the traffic light turns green. From the driver's seat, you may observe an object initially at rest in the truck bed spontaneously fly toward the tailgate, an obvious violation of the law of inertia! In contrast, an observer in the Earth frame of reference sees the object attempt to remain at rest as the truck accelerates forward, complying with the law of inertia.

Consider observers A and B with B moving at a *constant* velocity $\vec{v}_{AB}$ relative to A and each observing the same event, e. A measures the event at position $\vec{r}_{Ae}$, while B measures the same event at position $\vec{r}_{Be}$. According to Galilean relativity, we relate the two

$$\vec{r}_{Ae} = \vec{r}_{AB} + \vec{r}_{Be}, \tag{4-1}$$

where $\vec{r}_{AB}$ is the location of observer B relative to A. Assuming A and B perceive time the same (more on *that* assumption later!), we can write the following expressions:

$$\vec{r}_{Ae} = \vec{v}_{AB} t_e + \vec{r}_{Be} \tag{4-2}$$

$$\vec{v}_{Ae} = \vec{v}_{AB} + \vec{v}_{Be} \tag{4-3}$$

$$\vec{a}_{Ae} = \vec{a}_{Be}. \tag{4-4}$$

Assuming Galilean relativity is correct, then, an object's position and velocity would be *relative* quantities whereas the object's *change* in velocity would be an *invariant* quantity.

Learning Objectives

- Gain a qualitative understanding of the notion of reference frames
- Gain experience formulating a hypothesis analytically and testing it quantitatively

Exercise:

Part A – Experiment and Data Collection:

In this laboratory exercise, we want to observe the one-dimensional motion of an accelerating object from two different (approximately) inertial reference frames. Discussing with your lab partners, decide upon an accelerating object to observe. **Note:** *You are not restricted to objects within the laboratory*. Then, develop a way to observe the object from two different inertial reference frames. For simplicity, select the "Earth" frame of reference as one of your choices. The second frame must be moving (approximately) at a constant velocity relative to Earth. Two requirements:

1. Be safe!
2. Have fun with it!

While the accelerating object is in motion…

- For at least 8 different instances, record the time and position of the object from the perspective of the two frames of reference. Take care to use the *same* 8 instances for the two reference frames. In other words, the position values will be different in the two frames, but you should be observing the same 8 "events" in the two frames.
- Measure and record the positions of the "non-Earth" reference frame for 8 different instances. These need not be the same instances as the accelerating object.

On your lab report, briefly describe your method, using both words and a sketch.

Part B – Data Analysis:

Using the data from your spreadsheet, create graphs of the object's position vs. time, velocity vs. time, and acceleration vs. time from both the Earth and "non-Earth" frames of reference. On your report form, briefly note how the qualitative behavior of the graphs change between the two reference frames. Explain why the behavior does or does not make sense to you.

Use the object position vs. time data for the two reference frames to extract a value for the relative velocity of the two frames. There is more than one right way to do this! Discuss with your lab partners and together decide on the "best" way for your experiment.

Finally, use the direct measurements of the "non-Earth" reference frame position vs. time to extract a value for the relative velocity between the two frames. Again, discuss with your lab partners to decide on the best way to do this.

Part C – Interpretation of the Data:

Are the results of your data analysis consistent with what you expect from Galilean relativity? For example, is the behavior of your data consistent to what is predicted in Eqs. 4-2, 4-3, and 4-4. Are the values you extract in Part B for the relative velocity of the two frames consistent between the two methods? Justify your conclusions quantitatively. Do any ***systematic uncertainties*** complicate your ability to draw a conclusion? Justify your statements by citing quantitative measures.

Turn It In

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Lab 5. Interactions and Potential Energy

Overview

A change in the motion of an object generally indicates an interaction has taken place. Understanding the interactions between objects is critical to understanding the world around us and wielding that knowledge for practical applications. Often, if the motion of an object changes the kinetic energy also changes. One way to help us understand interactions is to classify them based on the way energy changes form. If the interaction irreversibly changes kinetic energy, e.g. to thermal energy, we say the interaction is "dissipative." Inelastic collisions are a common example. If kinetic energy is changed in a fully reversible process, we say the interaction is "nondissipative." An object in free-fall is a common example.

The reversibility of a process can be related to how the configuration of the interacting objects is changed. For instance, when the atoms in an object are reconfigured in an ordered or "coherent" way, process is reversible; and the form of energy associated with the change is potential energy. On the other hand, if the atoms are reconfigured in a random or "incoherent" way, the process is irreversible; and, typically, some kind of thermal energy is associated with the change. It is worth noting that thermal energy is also associated with the random or incoherent motion of atoms in an object. Contrast this with kinetic energy that is associated with the coherent motion of an object.

For an object in free-fall, kinetic energy will be exchanged for gravitational potential energy. Near Earth's surface, the change in an object's gravitational potential energy can be written

$$\Delta U^G = mg\Delta x; \qquad (5\text{-}1)$$

where m represents the object's mass and Δx the change in vertical position. Notice that this expression includes a change in the object's proximity to Earth, a *coherent* change in the configuration. When you consider other forms of potential energy, e.g. the energy stored in a spring, you might expect to find similar terms, e.g. one depending on the change in length, Δl. One sensible hypothesis is a general expression of the form

$$\Delta U = C(\Delta l)^n. \qquad (5\text{-}2)$$

C is a proportionality constant; and n is a unitless number, both of which can be determined experimentally.

Learning Objectives

- Gain a qualitative understanding of the various classes of interactions and energy
- Gain an introduction to analytical techniques to determining empirical formulae

Exercise:

Part A – Experiment and Data Collection:

In this laboratory exercise, we want to observe an approximately coherent exchange of energy between three different forms: kinetic energy, elastic potential energy, and gravitational potential energy. This will serve as an example of a nondissipative interaction. Discussing with your lab partners, develop a setup that will use an elastic process to put an object into vertical free-fall. **Note:** *You are not restricted to objects within the laboratory*. Feel free to discuss with your lab partners beforehand to make use of contraptions at your disposal. Some possible options

- Launch a small rubber ball with a rubber band or sling shot
- Catapult a small rubber ball with a plastic spoon
- Fire a compact wad of paper upward with a spring

As always, be as creative as you can be keeping it safe and fun. Bear in mind that you want to keep the projectile moving as close to straight up and down as you can. This will take some teamwork to calibrate. Give a brief description of your method on the lab report.

After aligning, calibrating, and testing your setup, perform eight different runs with your device. In each run, launch the object using your elastic device, deforming it so that the launched object achieves a different height each time. For example, if you use a rubber band, stretch it a different amount each time. In each run, record the amount by which you deformed the elastic device, i.e. the Δl value, and how high the object rose.

Part B – Data Analysis:

Use your spreadsheet to perform the requisite calculations to convert the launch heights into changes in gravitational potential energy. Take care to use the appropriate distance, i.e. measuring from the release point to the highest point on the trajectory.

If this is a nondissipative interaction, the gravitational potential energy at the highest point of the trajectory should equal the elastic potential energy at the lowest point of the trajectory, i.e. at the instant you set the object loose to be launched. We can use these calculated potential energy values to derive an empirical expression for elastic potential energy, e.g. to determine the value of n from Eq. 5-2. Discuss with your lab partners what value you expect and record this value with a short explanation on your report form.

Make an Excel plot of U vs. Δl. On your lab report, briefly discuss the qualitative dependence on Δl, i.e. is it constant, linear, faster than linear, etc.

Let's try a very common analysis trick that enables us to extract the power-dependence through a *linear* regression. Take the log of both sides of Eq. 5-2 and notice what happens to the n value! Use your spreadsheet to perform the requisite calculations and make a plot of $\log(U)$ vs. $\log(\Delta l)$ with a trendline. Use the linear regression capabilities of Excel to report an empirical value of n with its uncertainty.

Part C – Interpretation of the Data:

Are the results of your data analysis consistent with your expectation? For example, is $\log(U)$ vs. $\log(\Delta l)$ linear? Does the extracted n value make sense? Justify your conclusions quantitatively. Do any ***systematic uncertainties*** complicate your ability to draw a conclusion? Justify your statements by citing quantitative measures.

Turn It In

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Lab 6. Forces and Equilibrium

Overview

When an object's motion changes, we typically conclude an interaction has taken place. We define the force exerted on an object as the time rate of change in the object's momentum. Generalizing to the case of multiple interactions, we define the net force on an object as

$$\sum \vec{F} \equiv \frac{d\vec{p}}{dt}. \tag{6-1}$$

When the inertia of the object is constant, we recover the familiar expression $\sum \vec{F} = m\vec{a}$. When the translational motion of an object is unchanged, we say the object is in translational "equilibrium." The mathematical statement for such an equilibrium, then, is $\sum \vec{F} = 0$.

When a spring is displaced a relatively small distance from its equilibrium or "relaxed" length, the force exerted by the spring (s) on the load (l) is given by Hooke's law

$$F_{\mathrm{sl}x} = -k(x - x_0). \tag{6-2}$$

In Eq. 6-2, x_0 is the equilibrium length; and k is the so-called "spring constant." Hooke's law turns out to be a profoundly useful concept, not only for springs but for other systems displaced a small amount from stable equilibria. The "narrative" of Hooke's law is that to a first-order approximation, the restoring force near stable equilibrium depends *linearly* on the displacement from equilibrium. The "spring constant" represents a sort of "slope" for this linear dependence.

Learning Objectives

- Gain experience solving equilibrium problems for an example set of systems
- Gain experience with analytic techniques to analyze experimental data
- Gain an introduction to uncertainty propagation in calculated quantities

Exercise:

Part A – Experiment and Data Collection:

In this exercise, we want to analyze spring-mass systems at equilibrium. Your group will need to choose two springs, which you may either bring from home or find in the laboratory. You may choose any springs you wish. To get the smoothest experience in lab, you may want to select springs that are stiff enough to support a range of weights while still stretching enough for you to measure with something like a ruler. Outside of those considerations, be creative!

First, set up your springs individually. Suspend a spring vertically and hang a mass on the end, gently letting the spring stretch until it reaches equilibrium. Record the mass and the corresponding length stretched in a spreadsheet. Take 8 such measurements for each spring. Do not to hang such heavy masses that you deform the spring ("small" displacements!).

Next, repeat the exercise but suspend the springs side-by-side ("in parallel") so that the masses are supported by both springs. In your report, argue why this should be a "stronger" or "weaker" system than an individual spring. Take 8 measurements, recording the masses and lengths stretched in your spreadsheet. In principle, the springs should stretch the same amount, here; but you may need to work with your setup a bit to ensure this is the case.

Finally, repeat the exercise, again, but hook one spring onto the end of the other ("in series") and hang the masses from the bottom spring. In your report argue why this should be a "stronger" or "weaker" system than an individual spring. Again, take 8 measurements, recording the masses and the amount each spring stretches.

Part B – Data Analysis:

Use your spreadsheet to convert your mass data into applied forces on the springs. Then, using Eq. 6-2 as your guide, perform a linear regression of your "individual spring" data to extract the spring constant of each spring. Include an appropriate plot with trendline for each. Record the spring constants and their uncertainties on your lab report form.

Now, let's repeat our analysis on our "spring-pair" data. Can you plot the force and stretch length data in such a way that you get a linear relationship? If so, this will allow you to extract an "effective" spring constant for your spring-pair system. Discuss with your lab partners (and TA/instructor, as needed) and see if you can do it. Record the effective constants and their uncertainties on your lab report form.

Part C – Interpretation of the Data:

Give a brief discussion of whether your data conform to your expectation based upon Hooke's law. Make sure to point to quantitative measures from your regression analysis. Then, discuss the "effective" spring constant values. Do they make sense, based upon your individual spring constants? From Hooke's law and your skills solving equilibrium problems, work out theoretical expressions for the effective spring constants. Do the values you measure from the spring-pair data agree with those you expect from the individual values?

Turn It In

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Lab 7. Forces and 2-D Motion

Lab designed by Beth Jennings, ACU class of '22

Overview

In this lab, you're going to be utilizing your understanding of forces and two-dimensional (projectile) motion to estimate the amount of force you or one of your group members can exert with a punch, and then to calculate that value experimentally. The laboratory contains a variety of tools with which to do this, and your write-up gives you guidance as to what order you should perform certain tasks or calculations. Ultimately the way in which you complete these tasks is up to you. Feel free to experiment within the bounds of the lab, and try multiple approaches until you find one that works the best.

Learning Objectives

- Apply knowledge of forces and projectile motion to practical situations
- Gain experience with analytic techniques to analyze experimental data

Exercise:

Part A – Experiment and Data Collection:

Designate a group "puncher." First, punch an object horizontally so that it strikes a wall (and not the floor…or your lab partners!). With your group develop a method to estimate the amount of force exerted by this punch and test it with a few trials. Once settling upon a robust method, have the puncher perform four punches, recording in your spreadsheet the necessary information to estimate the force. Using this information, record in your lab report the best estimate of the force with an appropriate uncertainty.

Use the force value to estimate how far the object should travel horizontally given a horizontal punch, and record this estimate with its uncertainty in your lab report. Then, perform three horizontal punches, recording the horizontal distances traveled in your spreadsheet. Report the average distance traveled with its uncertainty in your lab report.

Finally, let's try something a bit more fun! Have your puncher execute a horizontal punch from an inertial reference frame moving relative to the Earth frame, e.g. in a rolling chair, on a skateboard, running steadily, etc. Take the measurements necessary to calculate the relative speed of the "moving" frame and record them in your spreadsheet. Also, record the horizontal distance traveled by the object as measured from the Earth frame of reference.

Part B – Data Analysis and Interpretation:

On your lab report form, discuss whether your predicted horizontal distances traveled for the object are consistent with your expectations based upon the estimated force of the punch and what you know about objects in free-fall. Give appropriate quantitative measures to justify your conclusions, in particular making note of the uncertainties involved. Discuss possible systematic uncertainties that may lurk and argue why they are relevant or not for your analysis. Again, base your conclusions on quantitative measures from the analysis of your data.

Turn It In

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2. Each student should hand in an independent Lab Report page, including a discussion of the results. Don't forget to sign your name at the top of the page!

Lab 8. Angular Momentum

Overview

The larger the momentum, $\vec{p} = m\vec{v}$, of an object, the more easily it can set another object into translational motion. You may well speculate that there is an analogous quantity related to the rotational motion of an object. We arrive at such a quantity by taking our expression for momentum and substituting the rotational analogues for inertia and velocity, namely *rotational* inertia (I) and *angular* velocity ($\vec{\omega}$). We call this quantity "angular momentum":

$$\vec{L} = I\vec{\omega}. \tag{8-1}$$

First, consider a particle of inertia m rotating from a distance r around an axis. The rotational inertia of the particle can be expressed $I = mr^2$ (more on rotational inertia in Lab 9). The rotational velocity can be expressed $\omega = v_t/r$, where v_t is the tangential component of translational velocity. Using these, the magnitude of angular momentum can be written

$$L = rmv_t. \tag{8-2}$$

Of course, objects that are *not* rotating can also set other objects into rotation! For example, if you throw a baseball that hits an open door, the door will start to swing on its hinges. For an object under translational velocity $\vec{v}$ at a perpendicular distance of $r_\perp$ to an axis of rotation, Eq. 9-2 can be equivalently rewritten

$$L = r_\perp mv. \tag{8-3}$$

Finally, like translational momentum, angular momentum is a *conserved* quantity. Therefore, in an isolated system, the total angular momentum—be it from rotating or translating objects—will remain unchanged. For example, consider two objects: object 1, stationary and free to rotate around a fixed axis, and object 2 of inertia m, traveling in a straight line at speed v with a perpendicular distance $r_\perp$ from the rotation axis. Initially all angular momentum lies with object 2, i.e. $L = r_\perp mv$. What if the objects collide inelastically? After the collision, all angular momentum lies with the rotating system containing both objects, i.e. $L = I\omega$. For our scenario, conservation of angular momentum requires

$$I\omega = r_\perp mv. \tag{8-4}$$

Consequently, if angular momentum is conserved, we should observe a *linear* relationship between the "projectile" speed and the resulting angular velocity of the rotating system.

Learning Objectives

- Gain a hands-on understanding of the different forms of angular momentum
- Practice setting up a simple experiment and analyzing the data collected

Exercise:

Part A – Experiment and Data Collection:

In this exercise, we want to test the prediction seen in Eq. 8-4 from conservation of angular momentum. With your group, design an appropriate apparatus for the inelastic collision. Use whatever objects you wish, from home or in the laboratory. Be creative! One simple option:

- Balance a stick or rod on the end of another such that it rotates freely. For example, plastic rulers often come prefab with a hole in the middle that can fit over a screw or pointed end of an aluminum lab table rod and rotate relatively freely.
- Attach small cups or baskets to the ends of the balanced rod as a means of catching the projectile object to ensure an inelastic collision.

To get the best results, consider "pre-balancing" your apparatus so that after the collision the system will rotate in the horizontal plane. Also, think about ways to reduce friction on the rotation joint. This will take some teamwork; and, yes, that's intentional!

Next, with your lab partners select a projectile to collide inelastically with your rotating device. Again, be creative and have fun with it! You may use anything you wish but make sure whatever you pick out is conducive to an inelastic collision.

Develop a method to induce inelastic collisions. Practice a few times to hone the process. Then, run at least five collisions. Take whatever measurements, videos, etc., you need to obtain the projectile velocity and angular velocity of your rotating system. Record the raw information in a spreadsheet and perform the calculations in your spreadsheet, as well.

Part B – Data Analysis:

Using your spreadsheet, perform the requisite calculations and make a plot of ω vs. v for your five collisions. Include a trendline on your plot. Finally, perform a linear regression of ω vs. v, recording on your lab report the slope, intercept, and R^2 for the regression.

Part C – Interpretation of the Data:

On your lab report, argue whether your data are consistent with the expectation of Eq. 8-4? Are systematic uncertainties relevant? Justify all statements with quantitative measures.

Turn It In

1. E-mail to your T/A, instructor, and lab partners a copy of your spreadsheet with the results of this laboratory. Don't forget to include the error analyses, regression output, associated graphs, etc., needed to obtain your results.
2. Each student should hand in an independent Lab Report page, including a discussion of the results. Don't forget to sign your name at the top of the page!

Lab 9. Rotational Inertia

Overview

We define the concept of “inertia” as the capacity of an object to resist a change to its translational motion. Inertia is an important component of the quantity, translational momentum. We extend this concept to rotations by introducing the quantity “rotational inertia,” the capacity of an object to resist a change to its *rotational* motion. You may easily surmise that an object’s rotational inertia depends upon how much inertia an object contains. However, it also depends upon how that inertia is *distributed* throughout the object. For a “point particle” of inertia m, rotational inertia is defined

$$I = mr^2 \tag{9-1}$$

where r represents the distance from the object to the axis of rotation. For an “extended” object, we can divide up the object into point-like chunks of rotational inertia $dI = dmr^2$ and integrate over all chunks. If we know an object’s rotational inertia for an axis passing through the center of mass, I_{cm}, we can calculate the rotational inertia through any axis parallel to this one by using the “parallel axis theorem”

$$I = I_{\text{cm}} + md^2 \tag{9-2}$$

where d represents the separation distance between the rotation axis and the center of mass.

Often, we are interested in knowing the rotational inertia of a spherical or cylindrical object, e.g. with inertia M and radius R, with a rotation axis passing through the center of mass. The different shapes of these objects will yield different rotational inertias. A simple and convenient way to characterize an object is by defining the “shape factor”

$$c \equiv I/MR^2 \tag{9-3}$$

Note: c is unitless and independent of an object’s particular inertia and dimension.

Changing the translational motion of an object with inertia requires a force; and the work done by this force changes the object’s energy, e.g. its kinetic or potential energy. Analogously, changing the rotational motion of an object with rotational inertia requires a torque. The work done by this torque can, e.g. change the object’s rotational kinetic energy. For an object undergoing both translational and rotational motion, a complete description involves both the translational and rotational kinetic energies.

Learning Objectives

- Gain a practical, hands-on understanding of rotational inertia
- Practice data analysis techniques
- Apply the equations of rotational kinematics

Exercise:

Part A – Experiment and Data Collection:

In this lab exercise, you will estimate the rotational inertia of an extended object, e.g. using the equations that define rotational inertia, and then measure the rotational inertia of this object using the concepts of rotational kinematics. You and your lab partners may choose whatever object you wish, so long as it has non-zero rotational inertia. A couple of options:

- One for which it will be easy to estimate the rotational inertia, e.g. an object resembling those in the table of extended objects in Table 11.3 of your textbook
- One for which it will be easy to measure the rotational inertia, e.g. an existing piece of lab equipment in the stockroom

With your lab partners, develop a method to estimate the rotational inertia of your object. For example, begin by identifying the shape in Table 11.3 that most closely resembles your object. You may then modify the equation based upon the reality of your object. Another option is to model your object as a combination of the shapes represented in Table 11.3, calculate the inertias using the appropriate equations for each, and add them together. You may document your calculations in a spreadsheet, on your lab report form, or both. In words, briefly describe your method and report your final estimate with an appropriate uncertainty on your lab report.

Next, with your lab partners develop a method to measure the rotational inertia of your object using the principles of rotational kinematics. One suggestion is to think of analogies to the lab exercises we did with translational motion. Another is to draw from scenarios discussed in example problems during lecture. Whatever method you develop, you must use the equations of rotational kinematics to relate the object's rotational inertia to the quantities you measure. Develop a method, do a trial to work out the kinks, then run it at least three times, collecting data. Most importantly, be creative and have fun!

Part B – Data Analysis:

Compile the data from your runs into a spreadsheet. With your lab partners, develop a way to turn your data into a "measurement" of the rotational inertia. Consider using techniques such as linear regressions or averages of several calculated values. Create a spreadsheet graph, if appropriate. Don't forget to include with your answer a sensible estimate of your experimental uncertainty! You may calculate and report this answer on your spreadsheet but please also report it with appropriate units and significant figures on your lab report form.

Part C – Interpretation of the Data:

Give some words to the story your data tell. Are the two results for your rotational inertia consistent? Cite a measure to justify your conclusion. Evaluate your data for the kinematics portion. Do they behave as you expect? For example, if you take the average of several values, how consistent are those values? If you use a linear regression, how linear are your data? With all of these, be *quantitative*! Wherever you draw a conclusion, cite a measure as justification. What ***systematic uncertainties*** could exist for your analysis? Are any of them relevant? Again, justify your statement by citing a quantitative measure.

Turn It In

3. E-mail to your T/A, instructor, and lab partners a copy of your spreadsheet with the results of this laboratory. Don't forget to include the error analyses, regression output, associated graphs, etc., needed to obtain your results.
4. Each student should hand in an independent Lab Report page, including a discussion of the results. Don't forget to sign your name at the top of the page!

Lab 10. Torque

Overview

By now, we know that forces produce translational acceleration. The rotational analogue is "torque," which produces rotational acceleration around an axis. We relate torque to force as

$$\vec{\tau} = \vec{r} \times \vec{F} \tag{10-1}$$

$$\tau = r_{\perp}F = rF_{\perp} = rF\sin\theta. \tag{10-2}$$

The strength of the torque delivered by a given force is directly related to the size of the lever arm. Just as force is a vector quantity, so torque is a vector quantity. The direction of the torque can be found using the right-hand rule and lies along the resulting axis of rotation. We found in our study of translational motion that $\sum F = ma$. With rotational motion, we find

$$\sum \vec{\tau} = I\vec{\alpha}. \tag{10-3}$$

When the sum of torques on a system is zero, the system is in "rotational equilibrium." For systems with both zero net torque and net force, the system is in "mechanical equilibrium."

Consider a rod of unknown mass resting on a pivot. If the pivot is placed at the rod's center of mass, the lever arm for the gravitational force on the rod will equal zero. If this is the only force applied to the rod, the net torque will equal zero; and the rod should balance. If a weight is placed on the end of the rod, the pivot will need to be shifted so the gravitational force on the rod produces a torque to counter that of the weight, balancing the rod. The new pivot position will depend upon how the mass of the weight compares to that of the rod. If the rod is of an unknown mass, one could use the known mass of the weight and the observed position of the pivot to determine it. Sounds like a good lab exercise, huh?

Learning Objectives

- Gain a hands-on understanding of torque and rotational equilibrium
- Practice data analysis techniques

Exercise:

Part A – Experiment and Data Collection:

In this lab exercise, you will determine the mass of an unknown object by using your knowledge of torque and equilibrium. Select with your group an object to balance. As usual, you may use whatever you wish, either something from the lab or something brought from home. For a smooth lab experience, it is advisable to choose something that is pretty flat and relatively easy to balance, like a meterstick, aluminum rod, etc. Then, choose an appropriate pivot upon which to balance the system. Again, use whatever you'd like. Something as simple as your finger should work just fine. Something you might want to do, first, is to find the object's center of mass, e.g. balancing it on the pivot by itself. You might want to make note of this spot for later.

Now, choose five different weights of "known" mass (feel free to measure them on a scale). One at a time, place each weight somewhere on the object and find the new pivot position that puts the system back into rotational equilibrium. Record the mass of the weights and the associated pivot positions in your spreadsheet. Give a brief description of your setup in your lab report.

Part B – Data Analysis:

Turn your data into a measurement of the object's mass! With your group, use the principles of rotational equilibrium to work out a relationship between the (unknown) mass of the object, the mass of the weights, and the lever arms. Document this in your lab report. Use your spreadsheet to make any needed intermediate calculations and extract the object's mass, e.g. use a linear regression, take an average of several calculations, etc. Make an appropriate graph of your data. Record the mass and its uncertainty on your lab report.

Part C – Interpretation of the Data:

Now, make a direct measurement of the mass of your object and compare it quantitatively to what you extracted using torques and equilibrium. Are the two consistent with each other? Are your data otherwise consistent with what you expect, e.g. from the mathematical relationship you worked out in Part B? Remember to be *quantitative*! Wherever you draw a conclusion, cite a measure as justification. What ***systematic uncertainties*** exist for your analysis? Are any relevant? Again, justify your statement by citing a quantitative measure.

Turn It In

1. E-mail to your T/A, instructor, and lab partners a copy of your spreadsheet with the results of this laboratory. Don't forget to include the error analyses, regression output, associated graphs, etc., needed to obtain your results.
2. Each student should hand in an independent Lab Report page, including a discussion of the results. Don't forget to sign your name at the top of the page!

Tips on Experimental Errors and Data Analysis

Data

1) *Record **raw** **measurements on the spreadsheet***, not subsequent calculated values. In this way, you can more easily find mistakes if they occur. For example, record the measured length, width, and height of a cube rather than its calculated volume.
2) ***Use a spreadsheet to do your calculations***, rather than a hand-held calculator, so that errors can be fixed more readily. Be sure to label the spreadsheet adequately so that you understand it later.
3) Label all rows and columns (including units) on your spreadsheet.
4) Add comments to your spreadsheet, as necessary.
5) Do not round off your answers until the last step of the calculation. Rounding-off too soon can introduce undesirable errors. Your final calculated result should be expressed with the proper number of significant figures, based upon your experimental uncertainty.
6) Quote your results as (result) ± (uncertainty in this result, with units), e.g., "The measured acceleration was $2.21 \pm 0.14\ \mathrm{m/s^2}$."

Graphs

Graphs of data and calculated quantities must **always** have:

- *a label on the x-axi*s giving the quantity plotted and the physical units
- *a label on the y-axi*s giving the quantity plotted and the physical units
- an appropriate *legend* clearly distinguishing between sets of data, if more than one series (a set of data *or* a best-fit line) is plotted on the same graph
- a *figure title* relevant to that experiment (not "John and Dave's Data").

When you perform a linear regression analysis, you will obtain values for the slope of the line and the intercept of the line from the regression. In Excel, this output will look like

	Coefficients	*Standard Error*
Intercept	0.711245	0.021334
X Variable	23.14976	1.188314

where:

0.711245 is the numerical value of the intercept of the straight line, and,

23.14976 is the numerical value of the slope of the straight line.

The uncertainty in the intercept is 0.021334, and, the uncertainty in the slope is 1.188314.

To obtain a linear regression in Excel

Open any spreadsheet with (x, y) data. Under the **Data** tab, in the **Analysis** box, select **Data Analysis**, and then **Regression**. If the **Analysis** box does not appear, it means the data analysis tools are not yet loaded (notify your lab instructor). With the cursor in the box labeled 'Input Y Range', highlight the correct block for the dependent (y) variable on your spreadsheet. Then move the cursor to the box labeled 'Input X Range', and do the same for the independent (x) variable. Then click **OK** and look at your results.

Some useful rules:

1. **The reported uncertainty should have 2 significant digits only.**

   E.g. Excel calculates the standard deviation or "error" in a list of numbers to be $\sigma = 0.0123$ m. Reporting your error or uncertainty as

$\sigma = 0.0123$ m	is WRONG (too many sig. figs. given)
$\sigma = 0.012$ m	is CORRECT
$\sigma = 0.01$ m	is WRONG (too few sig. figs) except for a measurement from a device for which the uncertainty is known to one digit

2. **If none specified, then it is roughly indicated by the number of significant figures.**

   E.g. g is given as 9.802 m/s^2, what is the uncertainty in that number? Roughly 0.001 m/s^2.

3. **The number of significant figures given must be consistent with the uncertainty.**

   E.g. we have made a measurement of g and we know it only as well as ± 0.011 m/s^2. Therefore, writing

$g = 9.80234 \pm 0.011$ m/s^2	is WRONG (too many sig. figs. given).
$g = 9.802 \pm 0.011$ m/s^2	is CORRECT.

Experimental Uncertainties

Suppose that a quantity x is measured. The uncertainty of the measurement is denoted σ (the Greek letter "sigma"). The result is expressed as:

$$x \pm \sigma$$

Other common terms:

- fractional error $= \frac{\sigma}{x}$
- percentage error $= \frac{\sigma}{x} \times 100\%$
- z-value $= \frac{|x_{\text{measured}} - x_{\text{expected}}|}{\sigma}$

Example: "I measured Mary's height with a meter stick to be 1.542 ± 0.005 m. The percentage error in the measurement is $0.005/1.542 \times 100\% = 0.3\%$."

If the difference between your measured value and the expected value exceeds 2σ, i.e. $z > 2$, you should attempt to account for the reason.

Types of Errors:

1. Systematic error: These are errors that affect all measurements in the same way or in some regular manner and are usually issues involving the **measuring instrument** or **details left out of the theory** guiding the analysis. Systematic error affects the **accuracy** of the measurement.

2. Random Error: If a measurement is repeated several times, slightly different values will be found, some higher than average and some lower. This type of uncertainty usually results from the **observer's judgment**. Random error affects the **precision** of the measurement.

There are several ways to estimate or determine the amount of random uncertainty.

1. Perform a linear regression with Excel
2. Take many measurements (5 – 10 or more) and calculate the standard deviation.
3. Take a few measurements, find the difference between the largest and smallest values, and set the uncertainty to half that difference.
4. Use the manufacturer-provided level of uncertainty, e.g. a scale labeled "$e = 0.01$ g," indicating that the uncertainty is 0.01 grams. If the uncertainty is not labeled on a device, assume it is half the smallest division, e.g. 0.0005 m on a ruler with 1 mm segments.

Identifying Systematic Error:

Assessing the amount of systematic uncertainty is less clear than with random error. When performing experiments that have an expected result, the "z-value" can suggest the presence of systematics. Typical sources are:

1. Erroneous Calibration: Because the calibration is applied to every single data point in the same way, all the data are shifted too high or too low.
2. Simplifying Assumptions in the Theory: When performing exacting experiments, approximations can lead to theories that do not adequately describe real-world results, e.g. neglecting air-resistance to simplify equations of motion.

Interpreting Results with Errors

When the same experiment is done many times, the repeated measurements will usually distribute themselves according to a Gaussian function (also known as a "bell-shaped curve"). Such a distribution can be characterized by its mean (or "average") and its "standard deviation" or σ, describing the width. Approximately 68% of repeated measurements will fall within one σ on either side of the mean, approximately 95% will fall within 2σ, and 99.7% within 3σ.

If calculated correctly, the error obtained using the techniques described previously gives an estimate of the standard deviation of numerous repeated trials. You can compare your result to an expectation by calculating how many σ the result differs from the expected result. The difference between the measured and expected values is the "z-value," expressed as

$$z = \frac{|\text{measured} - \text{expected}|}{\text{uncertainty}}$$

An example is the following:

Measured value of charge on an electron: $Q = 1.573 \pm 0.018\ \times 10^{-19}$ Coulombs (C)

Expected value: 1.602×10^{-19} C

$$z = (1.602 \times 10^{-19} - 1.573 \times 10^{-19})/(0.018 \times 10^{-19}) = 1.6$$

The result of one experiment does not "prove" anything. However, the custom in physics is to interpret an experimental result less than 2σ from expectation as "**consistent** with the expected value." For the example above, you can write a conclusion as follows:

"My experimental result of $1.573 \pm 0.018 \times 10^{-19}$ C is 1.6 standard deviations away from the expected value of 1.602×10^{-19} C and is therefore consistent with the expected value."

Results more than 2σ away from the expected values may suggest more than just random uncertainties are involved in the experiment. If this happens, report your results and attempt to list possible factors that could cause systematics. Systematic errors often exist due to incorrectly calibrated equipment, incorrect assumptions, or poor experimental technique. If a result is more than 3σ from expectation is typically said to be "**inconsistent**" with the expected value.

Propagation of Errors (Combination of Errors)

When measured quantities are added, multiplied, etc., then their uncertainties must be combined in some way to find the total uncertainty in the result. The full rules about how to do this are somewhat complex (see your instructor if you wish to know more). There are some simple rules and approximate methods shown below that can be used in this course. In the examples, we'll use x and y as the measured values with uncertainties σ_x and σ_y, respectively.

Simple Rules

1. **Addition or subtraction:** The uncertainties add like in Pythagorean's Theorem.

$$z = x + y \text{ or } z = x - y \quad \rightarrow \sigma_z = \sqrt{\sigma_x^2 + \sigma_y^2}$$

2. **Multiplication or division:** The fractional uncertainties add like in Pythagorean's Theorem.

$$z = xy \text{ or } z = \frac{x}{y} \quad \rightarrow \frac{\sigma_z}{z} = \sqrt{\left(\frac{\sigma_x}{x}\right)^2 + \left(\frac{\sigma_y}{y}\right)^2}$$

3. **Exponent:** The fractional uncertainty gets multiplied by the exponent.

$$z = x^b \rightarrow \frac{\sigma_z}{z} = b\frac{\sigma_x}{x}$$

Example:

You want to know the area of a table top. You measure the lengths of the sides to be $l = 2.307 \pm 0.005$ m and $w = 1.34 \pm 0.01$ m (perhaps the edges in this direction are more rounded). The area is $A = lw = 3.09\ m^2$. The uncertainty would be

$$\frac{\sigma_A}{A} = \sqrt{\left(\frac{\sigma_l}{l}\right)^2 + \left(\frac{\sigma_w}{w}\right)^2} = \sqrt{\left(\frac{0.005}{2.307}\right)^2 + \left(\frac{0.01}{1.34}\right)^2} = 0.008 \rightarrow \sigma_A = (3.09\ \mathrm{m}^2)(0.008) = 0.02\ \mathrm{m}^2$$

Significant Figures:

To indicate the precision of a result, we write a number with as many digits as are "significant." The number of significant figures in a number is defined as follows:

1. The leftmost non-zero digit is the most significant digit.
2. If there is no decimal point, the rightmost non-zero digit is the least significant digit.
3. If there is a decimal point, the rightmost digit is the least significant digit, even if a 0.
4. All digits between the least and most significant, inclusive, are counted as significant.

E.g. The following numbers each have four significant digits:

1234, 123400, 123.4, 1001, 1000., 10.10, 0.00001010

- In recording the result of a measurement or calculation, **one and only one** doubtful digit is retained. E.g. measuring a length with a meter ruler where the smallest scale division is 1 mm. A typical measurement would read 58.6 mm, since you can judge the measurement to tenths of a mm but not to hundredths.

- In addition or subtraction, do not carry the result beyond the first column which contains a doubtful figure.

2807.5	(tenths are the doubtful digit)
0.065	(thousandths are the doubtful digit)
+ 83.79	(hundredths are the doubtful digit)
2891.355	

Round off at the end of the calculation. So, the sum is correctly expressed as 2891.4

Note that in many cases of subtraction, the final answer has fewer significant figures than either of the original numbers.

49563	(5 sig. figs.)
-49532	
31	(2 sig. figs.)

- In multiplication or division, carry the result only to the same number of significant figures as there are in that quantity in the calculation which has the fewest significant figures.

  E.g. $10.51 \times 0.041 = 0.43091$

  Since the quantity with the fewest significant figures is 0.041 (2 sig. figs.) then the answer should only be given to **two** significant figures, i.e. as 0.43.

- Round off your estimates of errors to two significant digits.
- **When dealing with measured values**, where the uncertainty has been estimated or determined, **match the final answer's number of significant digits to the level of uncertainty, so the final digit in both the value and the error is in the same position with respect to the decimal.**

  E.g. if the uncertainty is 0.013 m/s and the value is measured to be 2.3457 m/s, the correct representation is 2.346 ± 0.013 m/s.