Chaos in Context: The Lorenz System

Project Brief

**Audience:** Other students with your level of experience with topics in calculus and basic ODEs.

**Due Date:** This assignment is due by 11:59 PM, Tuesday November 27.

**Resources:** You should use your textbook, *Section 9.8: Chaos and Strange Attractors*, as well as the provided reading *Chaos in Context* as references for this project. Your final screen cast should include one additional resource from a published source (i.e. textbook, journal article, magazine article etc...).

In 1963 Edward Lorenz explored a problem involving thermal convection. The result of this pursuit is the nonlinear system of equations:

\[
\begin{align*}
\frac{dx}{dt} &= \sigma(y - x) \\
\frac{dy}{dt} &= rx - y - xz \\
\frac{dz}{dt} &= xy - bz 
\end{align*}
\]

For very carefully chosen values of the parameters \(\sigma, r, b\) the solution to the system is called *chaotic*.

**Definition 1 (Chaotic System).** A system of differential equations is chaotic if the system is sensitive to initial conditions. This means small changes in the initial condition yield widely differing behaviors for the solutions. Such models are highly unreliable and very difficult to predict and are called chaotic.

The Lorenz system exhibits a set of points to which solutions will be attracted over time. This attracting set is called the *Lorenz attractor* and is in fact one of the primary examples of a *strange attractor*. This project will rely heavily on your knowledge of nonlinear systems and bifurcations, specifically those ideas developed in Sections 9.1-9.3 and 9.8 of the text. Some ideas your group may want to consider:

- Solve the Lorenz system for a variety of parameters explaining each in context. Walk through a careful exploration of the linearization of the model.
- The Lorenz system is a dissipative system with no possibility of a limit cycle in certain cases. Explain the significance of these statements.
- The article, “Chaos in Context” introduces the Lorenz model as a model which describes certain atmospheric conditions. In what other contexts has this model been utilized? Give examples of these models, and solve a few explaining them in context.
- The Lorenz system exhibits what is known as a Hopf bifurcation. Discuss the critical point nature of the linearized system via Hopf bifurcations.
- The Lorenz system’s chaotic solutions actually lead into a discussion of fractals. Explain the link between chaos and fractals in light of this example.